

NATIONAL BUREAU OF STANDARDS REPORT

10 498

SEQUENCING THE PURCHASE AND RETIREMENT OF FIRE ENGINES

Prepared for

The Fire Research Program
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U.S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

NATIONAL BUREAU OF STANDARDS

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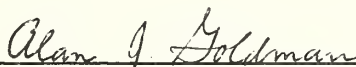
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SEQUENCING THE PURCHASE AND RETIREMENT OF FIRE ENGINES

1. INTRODUCTION

This report describes a method to determine an "optimum" manner of sequencing the purchase and retirement of fire engines (hereafter simply called "engines"), with specific application to the Washington, D. C. Fire Department. The model developed, however, has more general applicability as regards both the equipment type and the fire department. Because of the apparent similarity of the present problem to conventional equipment replacement problems, we first review in brief some of the ideas in the equipment replacement literature.

Equipment replacement problems have a long history in industrial engineering and operations research. The reader is referred to [8] for a comprehensive bibliography on this subject. One class of equipment replacement problems balances the cost of failures against the cost of planned replacements (see [3]). If units are to operate continuously over some time period $[0, t]$ and are replaced upon failure, then typically the expected cost $C(t)$ during $[0, t]$ may be given by

$$C(t) = c_1 E[N_1(t)] + c_2 E[N_2(t)], \quad (1.1)$$

where

c_1 = per unit total cost resulting from a failure and its replacement,

c_2 = per unit total cost of replacing a non-failed item ($c_2 < c_1$);

$N_1(t)$ = the number of failures in $[0, t]$, a random variable,

$N_2(t)$ = the number of replacements of non-failed units, a random variable,

and E denotes expected value. The problem is to minimize (1.1) over the possible replacement procedures available within a given policy of replacement. Examples of replacement policies are: strictly periodic replacement, random periodic replacement and sequentially determined replacement. Electronic components typify the equipment to which this well developed mathematical theory applies.

A second class of equipment replacement problems, called "preparedness" problems, assumes that a piece of equipment is kept in a readiness state for use in case of emergency. The objective is to maintain the equipment in a state of operational readiness at minimal cost. Thus a sequence of inspection and replacement actions that minimizes the ratio of expected cost per unit time to proportion of good time, would constitute an "optimal" decision stream (see [8], [10]). Large military hardware provides examples of the type of equipment to which this class of models may be applied.

One of the basic underlying concepts of the two classes of equipment replacement models discussed so far is that of a reliability

function¹. This is the probability $R(t)$ that the equipment is "good"² at time t (measured from a time at which the equipment is considered to be "new") and is exemplified by the negative exponential form

$$R(t) = \exp(-\lambda t). \quad (1.2)$$

A closely related concept is the failure rate, defined for any reliability function $R(t)$ as $\rho(t) = -R'(t)/R(t)$, where the prime denotes the derivative. For the negative exponential, the failure rate is the constant λ .

A third class of equipment replacement problems deals with the replacement of items that deteriorate. Mathematical models to solve this class of problems typically trade off the increasing operational and maintenance costs (and decreasing resale value) of an aging item against the cost of a new purchase, i.e., the "optimal" replacement time is that time at which these opposing forces are equalized. Dreyfus [6] used a dynamic programming approach to solve this problem under the additional complication of technological change.

The main concern of this report is the development of a model to determine purchase and retirement decisions over a planning period, subject to certain constraints, which would minimize the cost of

¹See [11] for a discussion of the statistical theory of reliability.

²It is implicitly assumed that the equipment is either in a "good" or a "failed" state.

operation of a fleet of engines during that period. The concern of the Washington, D. C. Fire Department was not with the cost of failure or the distribution of failures of fire engines per se, primarily because of the negligible number of engine failures and the inability to measure the "cost" of a single engine failure. The model developed may be regarded as an extension of the ideas represented by the third class of equipment replacement problems discussed above.

Section 2 describes a simple calculation, which serves to introduce the data at hand and compares the results of this calculation (as applied to Washington, D. C.) to those of a study [2] from which the data were obtained. A dynamic programming (DP) model is formulated and given illustrative application in Section 3, and directions for further investigation are suggested in Section 4. Appendix A develops certain details of the DP model and a listing of the DP computer code appears in Appendix B. Finally, an integer programming (IP) analog to the DP model is given in Appendix C.

2. INITIAL CONSIDERATIONS

Aside from personal communications with members of the staff of the Washington, D. C. Fire Department, the main source of data was a report by Balcolm [2]. This report also proposes a model, for determining the life-span of an engine, which will be described later.

A linear relationship between engine age and maintenance cost was used in [2], and least-squares regressions yielded three sets of coefficients, corresponding to "high usage," "medium usage," and "low usage" engines. Balcolm then obtained a "composite" equation-- a weighted average (by the number of engines in the three categories)-- which this report also uses. This equation is of the form:

$$u_a = U_0 + U_1 a, \quad (2.1)$$

where

a = engine age,

u_a = the maintenance cost of an engine entering its a^{th} year of service,

$U_0 = 24.17$,

$U_1 = 122.46/\text{year}.$ ³

Values of u_a are listed in Table 3.1. This relationship was adopted as the basis of the data for maintenance cost since it was felt that a more complex function could not be supported by the observed cost figures.

³All monetary quantities are expressed in dollars.

A linear relationship was also used in [2] for the purchase price of a new engine, given by

$$P_t = P_0 + P_1 (t - 1900), \quad (2.2)$$

where

$$P_0 = -16258.18,$$

$$P_1 = 576.87.$$

Values of P_t are given in Table 3.1. The choice of the "base" year 1900 is not explained, but it accounts for the surprising (negative) value of P_0 . The index t refers to the year for which a value of the purchase price is desired.

Using these data, a simple calculation can be made to determine an "optimum" life-span for a single engine. Assuming a zero salvage value (for simplicity)⁴ and a constant purchase price, the accumulated total cost of keeping an engine for n years is

$$\begin{aligned} TC(n) &= \sum_{a=1}^n (U_0 + U_1 a) + P \\ &= nU_0 + U_1 \sum_{a=1}^n a + P \\ &= nU_0 + [n(n+1)/2] U_1 + P. \end{aligned} \quad (2.3)$$

Thus the average annual cost of keeping an engine for n years is

$$\begin{aligned} AC(n) &= TC(n)/n \\ &= U_0 + [(n+1)/2] U_1 + P/n. \end{aligned} \quad (2.4)$$

⁴Constant salvage values (with respect to age) can be represented by subtracting them from U_0 .

Clearly, the longer an engine is kept, the longer the time to amortize the price P , so that portion of the cost per year will decrease with n . However, the maintenance costs increase year by year. Thus, with the "optimum" life-span defined as that value of n which minimizes (2.4), the standard calculus technique of setting the derivative of (2.4) to zero and solving for n yields:

$$(d/dn) (AC(n)) = U_1/2 - P/n^2 = 0, \quad (2.5)$$

whence

$$n = (2P/U_1)^{1/2}. \quad (2.6)$$

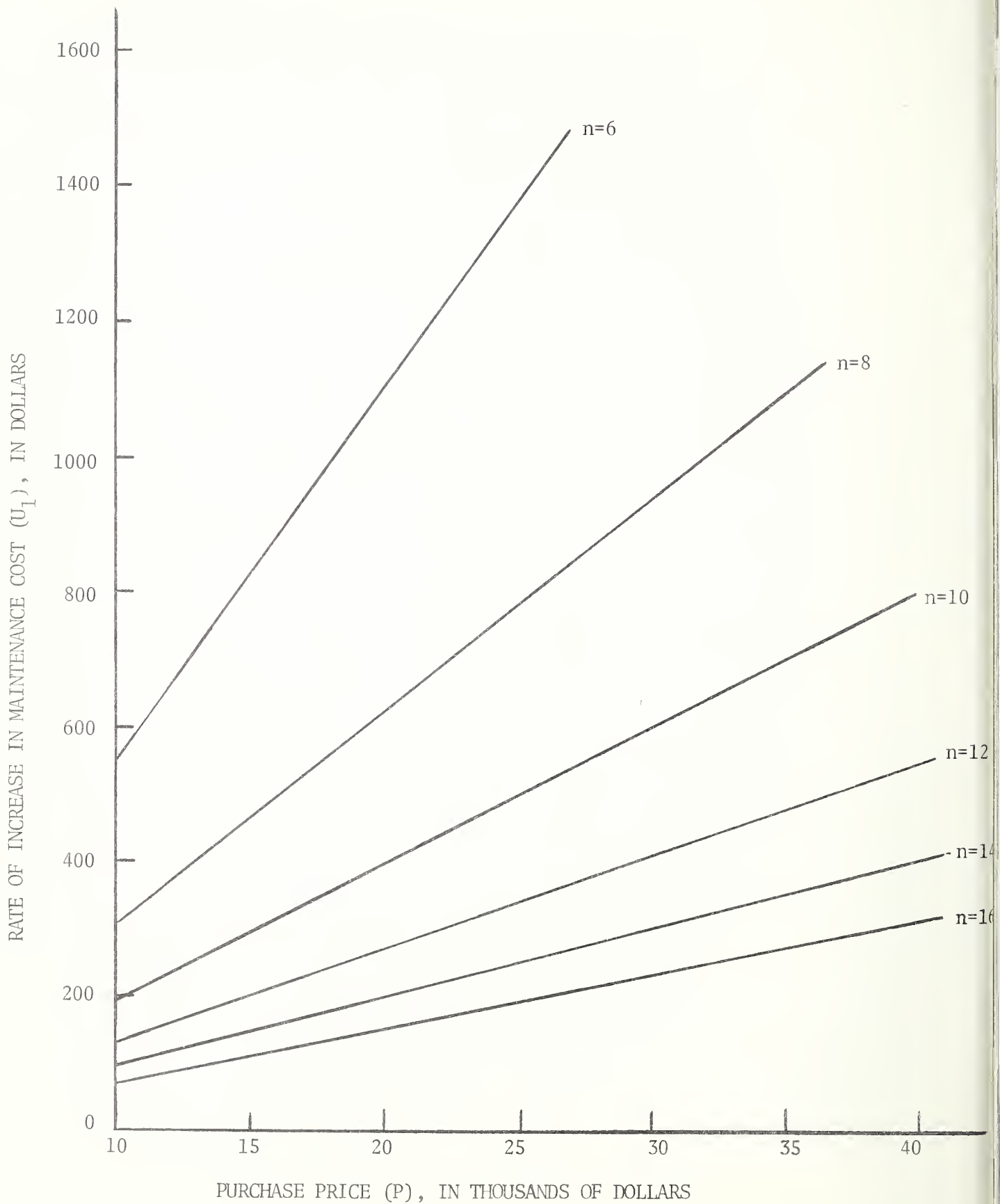
Since $P > 0$ and $n > 0$, the second derivative $2P/n^3$ is positive so that the value of n given in (2.6) ensures a minimum value of (2.4). Figure 2.1 indicates contours of the optimum value of n in the (U_1, P) -plane.

For Washington, D. C., using the 1969 purchase price, (2.6) yields $n = 19.6$, considerably larger than the present life span of 15 years. Balcolm [2] recommends a life span of 10-11 years, depending on the number of years over which an engine is linearly depreciated, using as his criterion the equality of current (resale) value and accumulated repair cost, i.e., n is chosen so that

$$P - n(P - S)/N = \sum_{a=1}^n U_a,$$

where N is the number of years over which an engine is depreciated and S is the salvage value of an engine after N years. (Note that Balcolm assumes that the number of years over which an engine is depreciated (N) and the number of years it is kept (n) need not

FIGURE 2.1 CONTOURS OF THE OPTIMUM ENGINE LIFE



be the same.) No rationale for this criterion is offered in [2], but the large difference between [2]'s 'optimum' life span and the one derived from the present calculation indicates a significant difference between the two models.

3. A DYNAMIC PROGRAMMING MODEL

The dynamic programming (DP) model described in this section takes a somewhat different approach to the problem of equipment replacement. Instead of determining an "optimum" life-span which would be applied to all engines, the DP model begins with the existing scenario and prescribes purchasing and retiring decisions over a T-year planning horizon. (The index $t = 1, \dots, T$ is used in this model and appropriate notation changes are made in the relevant formulas presented in Section 2.) In this sense, the model may be "tailored" to fit the initial state of affairs of any urban fire department. The reader interested in DP in general, is referred to the text [9]. For other DP formulations of equipment replacement problems, see [1] and [4].

In accordance with the concerns and objectives of the Washington, D. C. Fire Department, the DP model determines the purchases and retirements to be made during the planning horizon such that the total cost incurred during this period is minimized. The model accounts for various constraints within which a fire department must operate, e.g., constraints on the number of purchases and/or retirements which may be made in any year, the total fleet size, and the maximum allowable engine age.

The DP "state variables" (those which describe the system at each stage, or year in this case) are:

x_{1t} = the number of engines in the initial fleet which remain in year $t-1$,

x_{2t} = the number of new engines purchased in years $1, \dots, t-1$,

x_{3t} = the maintenance cost in year $t-1$ on engines purchased in years $1, \dots, t-1$.

(Note that $x_{1t} + x_{2t}$ is the fleet size in year $t-1$.) The "decision variables" are

d_{1t} = the number of engines retired from the initial fleet in year t ,

d_{2t} = the number of engines purchased in year t .

It should be emphasized that retirements are made only from engines in the initial fleet, i.e., none of the engines purchased during the planning period are considered for retirement. Since the Washington, D. C. Fire Department indicated interest in a planning horizon of at most five to ten years, restriction to retiring engines from the initial fleet only is not considered a limitation.

The data required by the model are :

D_t = the minimum number of engines required during year t (checked against the fleet size after year t 's decisions have been made),⁵

M_t = the maximum number of engines which may be purchased in year t ,

⁵That D_t adequately measures the demand for fire service is a simplification.

N_t = the maximum number of engines which may be retired in year t ,

R = the age by which engines must be retired,

P_t = the purchase price of a new engine in year t ,

Q_a = the number of a -year old engines in the initial fleet,

$m = \sum_a Q_a$ = the initial fleet size,

u_a = the maintenance cost of an engine during its a^{th} year of service,

v_{at} = the resale value in year t of an engine which was initially of age a ,⁶

a_i = the age of the i^{th} youngest engine in the initial fleet (e.g., a_1 is the youngest).

As in the simple model of section 2, the maintenance costs are calculated as

$$u_a = U_0 + U_1 a,$$

with the values of U_0 and U_1 , as indicated earlier. The linear relationship leads to a recursive definition of u_a ,

$$\begin{aligned} u_{a+1} &= U_0 + U_1(a+1) \\ &= U_0 + U_1 a + U_1 \\ &= u_a + U_1. \end{aligned} \tag{3.1}$$

Letting $x_t = (x_{1t}, x_{2t}, x_{3t})$, (3.1) may be used to obtain, as the stage transformation formula,

⁶The convention is adopted that an a -year-old engine in the initial fleet enters its $(a+1)^{\text{st}}$ year of service at $t=1$. It is assumed, for simplicity, that decisions are made at the beginning of a year, and that $a \geq 1$.

$$x_{t+1} = (x_{1t} - d_{1t}, x_{2t} + d_{2t}, x_{3t} + u_1 d_{2t} + U_1 x_{2t}). \quad (3.2)$$

The transformation for x_{1t} and x_{2t} is clear. The value of $x_{3,t+1}$, the maintenance cost in year t on engines purchased in years $1, \dots, t$, is obtained by adding to x_{3t} both the cost of the first year of maintenance for engines purchased in year t ($u_1 d_{2t}$), and the incremental increase in maintenance cost on engines purchased in the preceding years ($U_1 x_{2t}$), the latter deriving from (3.1).

The "stage return" is the cost of operation in year t . With the notation $d_t = (d_{1t}, d_{2t})$, the stage return is calculated as:

$$I_t(x_t, d_t) = (P_t + u_1) d_{2t} + \sum_{i=1}^{x_{1t} - d_{1t}} U_{a_i + t} - \sum_{i=x_{1t} - d_{1t} + 1}^{x_{1t}} v_{a_i + t} + x_{3t} + U_1 x_{2t}.^7 \quad (3.3)$$

The components of (3.3) have the following interpretations:

$(P_t + u_1) d_{2t}$ = the cost of purchasing d_{2t} engines in

year t and maintaining them during the

first year of service,

$\sum_{i=1}^{x_{1t} - d_{1t}} U_{a_i + t}$ = the maintenance cost in year t on engines which remain from the initial fleet,

$\sum_{i=x_{1t} - d_{1t} + 1}^{x_{1t}} v_{a_i + t}$ = the revenue from retiring the d_{1t} oldest engines not previously retired,⁸

⁷Whenever the lower limit of a summation exceeds the upper limit, the summation is taken to be zero. This is a standard notational convenience.
⁸This assumption of retiring "oldest" first" is supported by the Washington, D. C. Fire Department.

$x_{3t} + U_1 x_{2t}$ = the maintenance cost in year t on engines
purchased in years $1, \dots, t-1$.

The linear form of the maintenance cost yields the pleasing result that the values of x_{3t} are all exact multiples of U_1 .⁹ This, together with the fact that x_{1t} and x_{2t} are integers bounded by the constraints, makes it computationally feasible to consider all of the combinations of values that the state variables may assume in any stage. It follows that the optimal solution is exact, a condition not often found in DP problems. This characteristic is explicitly noted here as a favorable feature of the model.

The recursive equations of the DP model are:

$$\begin{aligned} f_t(x_t) &= \min_{d_t} [I_t(x_t, d_t) + f_{t+1}(x_{t+1})/(1+r)], \\ f_T(x_T) &= \min_{d_T} I_T(x_T, d_T). \end{aligned} \tag{3.4}$$

The quantity r is a discount rate, so that division by $(1+r)$ in the first relation of (3.4) renders $f_t(x_t)$ as the minimum present value cost of operations from years t through T , given that the state of the system in year t is x_t . Since the initial state is known to be $x_1 = (m, 0, 0)$, $f_1(m, 0, 0)$ is the optimal value of the objective, i.e., the minimum total cost of operations in years $1, \dots, T$.

The constraints of the DP model are straightforward from the definitions of the variables and parameters:

⁹This will be proven in Appendix A.

$$0 \leq d_{1t} \leq N_t \quad (t = 1, \dots, T), \quad (3.5)$$

$$0 \leq d_{2t} \leq M_t \quad (t = 1, \dots, T), \quad (3.6)$$

$$x_{1t} + x_{2t} \geq D_{t-1} \quad (t = 2, \dots, T+1), \quad (3.7)$$

$$\sum_{j=1}^{t-1} d_{ij} \geq n_t \quad (t = 2, \dots, T+1) \quad (3.8)$$

where $n_t = \sum_{a \geq R-t+1} Q_a$ is the number of engines which must be retired prior to year t because of the age limitation R . Note that by definition the initial conditions are: $x_{11} = m$, $x_{21} = 0$, $x_{31} = 0$, and $n_1 = 0$. With the definition $D_0 = m$, (3.7) and (3.8) automatically hold for $t = 1$.

The constraints (3.5) - (3.8) and the relationships among the state and decision variables lead to interesting and computationally useful results which are detailed in Appendix A. Suffice it to say here that a special computer code,¹⁰ developed as a part of this effort, takes advantage of these results to make it possible to solve larger problems than could be handled by a general purpose DP code. Furthermore, experience thus far has indicated that computer running times are significantly shorter using the special code. For example, one of the runs to be discussed below took 12 seconds using the special code, while the general purpose code¹¹ took 227 seconds.

¹⁰A listing of this code appears in Appendix B.

¹¹This code is an extension of the code documented in [5].

(Both codes are written in FORTRAN V and runs were made on the UNIVAC 1108 at NBS under the EXEC II Operating System.)

In exercising the DP model, the maintenance costs and purchase prices were the same as those discussed previously (cf., Section 2). The purchase price function was modified to

$$P_t = P_0 + P_1 (70 + t), \quad (3.9)$$

so that $t = 1$ would correspond to 1971. The values of P_0 and P_1 are unaffected by the modification and remain as listed under equation (2.2). The resale values v_{at} were calculated on the basis of (3.9), assuming an annual depreciation rate ρ , as

$$v_{at} = (1 - \rho)^{a+t-1} [P_0 + P_1 (70-a+1)], \quad (3.10)$$

so that resale values of engines in the initial fleet (purchased prior to $t = 1$) could be calculated from the appropriate purchase prices.¹² Finally, values of Q_a were obtained directly from the Washington, D. C. Fire Department's inventory of engines. These data are given in Table 3.1 with $T = 5$ (a five-year planning horizon).¹³

For the remaining data specifications, it was suggested by members of the Fire Department staff to take $R = 15$ (the present maximum engine age in Washington), $D_t = 64$ for $t = 0, \dots, 5$ (i.e. constant

¹²A geometric depreciation is not required by the model. It is incorporated in the code, but can easily be modified with minor coding changes.

¹³Members of Fire Department staff advised that a planning period of more than five years is unreasonable.

TABLE 3.1 - DATA FOR THE DYNAMIC PROGRAMMING MODEL

a	Q _a	u _a [*]	v _{at} [*]				
			t=1	t=2	t=3	t=4	t=5
1	4	147	14474	8684	5211	3126	1876
2	0	269	8477	5086	3052	1831	1099
3	10	392	4961	2977	1786	1072	643
4	5	514	2902	1741	1045	627	376
5	0	636	1696	1018	611	366	220
6	5	759	991	595	357	214	128
7	10	881	578	347	208	125	75
8	0	1004	337	202	121	73	44
9	5	1126	197	118	71	42	25
10	5	1249	114	69	41	25	15
11	3	1371	67	40	24	14	9
12	4	1494	39	23	14	8	--
13	4	1616	22	13	8	--	--
14	4	1739	13	8	--	--	--
15	5	1861	8	--	--	--	--

t	P _t [*]
1 (1971)	24700
2	25276
3	25853
4	26430
5 (1975)	27007

*Values have been rounded to the nearest dollar.

minimum required fleet size equal to the present fleet size), and $M_t = N_t = 6$ for $t = 1, \dots, 5$ (constant and equal purchase and retirement ceilings).

A base run was made with no discounting, i.e., $r = 0$, and the resultant "optimal" decisions were to purchase and retire 6 engines in each of the first three years and to purchase and retire 2 engines in year 4, i.e., $d_{1t} = d_{2t} = 6$ ($t=1, 2, 3$), $d_{14} = d_{24} = 2$, $d_{15} = d_{25} = 0$. Note from the age distribution Q_a in Table 3.1 that 20 engines reach the mandatory retirement age by year 5 (i.e., $n_6 = 20$). Since the maximum number of retirements permissible is 6 in each year, the optimal policy is to retire the 20 engines as soon as possible (ASAP policy), replacing them with new engines to meet the minimum required fleet size.

The above results are not surprising in view of the discount rate $r = 0$. Increasing maintenance costs, decreasing salvage values, and increasing purchase prices all indicate early retirement. The same policy is optimal in the extreme case where the purchase price is always zero. It is intuitively obvious that in this situation the ASAP policy is optimal regardless of the value of r , since the newly acquired (free) engines are operated at a lower maintenance cost than are the old ones.

In order to study the effect of the discount rate r on the optimal decisions, a series of runs was made with U_1 as a parameter, taken from 62.46 to 162.46 in increments of 10.00. [Recall that the "nominal" value of U_1 is 122.46.] Initially, r was varied from

0.0 to 0.5 in increments of 0.1 (a very rough grid), and based upon these results, smaller ranges with finer increments were studied for certain values of U_1 . The following observations were made consistently from the outputs of all the runs:

- (1) The only engines retired were the 20 which reach their maximum age during the 5-year planning period.
- (2) In every year, the numbers of purchases and retirements were the same. This may be attributable to the constant demand and to the constant and equal values of M_t and N_t over all t .
- (3) For those values of r considered, there was a value r_E such that for $r \leq r_E$ the ASAP policy was optimal, and a value r_L such that for $r \geq r_L$ the optimal policy was to retire as late as possible (ALAP policy) [The ALAP policy has $d_{11} = d_{12} = 5$, $d_{1t} = d_{2t} = 4$ ($t=2, 3, 4$), $d_{15} = d_{25} = 3$ for this particular problem.]
- (4) The values of r_E , r_L and $r_L - r_E$ are monotonically increasing functions of U_1 .

The values of U_1 for which the behavior of the optimal policy, as a function of r , was studied in greater detail are listed in Table 3.2 together with the relevant results. All other values of U_1 considered gave rise to values of $r_E = 0.0$ and $r_L = 0.1$ in the initial runs. It can be seen from Table 3.2 that the finest

TABLE 3.2 - RESULTS OF FINER VARIATION OF r FOR CERTAIN VALUES OF THE
PARAMETER U_1

U_1	Range of r	Increment	r_E	r_L
62.46	.01 - .10	.01	.05	.06
122.46	.08 - .09	.001	.080	.089
152.46	.01 - .20	.01	.09	.11
162.46	.01 - .20	.01	.10	.11

analysis with the smallest increments for r was made for the "nominal" value of $U_1 = 122.46$. For $.080 < r < .089$ the optimal decisions were "mixed", i.e., neither an ASAP nor an ALAP policy. For example with $r = .085$, the optimal decisions were

$$\begin{aligned} d_{11} &= d_{12} = 5, & d_{12} &= d_{22} = 6, \\ d_{13} &= d_{23} = 6, & d_{14} &= d_{24} = 3, \\ d_{15} &= d_{25} = 0. \end{aligned}$$

The "critical" range of r (.080, .089) is quite small, but it should be noted that the values $M_t = N_t = 6$ do not permit a drastic difference between the ASAP policy and the ALAP policy.

It is clear that if a value of r is specified, then the DP model may be run to determine the optimal policy. If r cannot be specified, then the values of r_E and r_L may be determined for a given value of U_1 . Then one need only specify whether $r \leq r_E$ or $r \geq r_L$ to conclude that the ASAP policy or ALAP policy, respectively, is optimal.

One run was made with $M_t = N_t = 10$ for all t and the other data remaining the same. With $r = 0$, the ASAP policy resulted; in this case $d_{11} = d_{12} = d_{21} = d_{22} = 10$, $d_{1t} = d_{2t} = 0$ ($t = 3, 4, 5$). Unfortunately, lack of time prevented further study of this case. Intuitively, one might expect a greater "critical" range of r since the larger values of M_t and N_t given rise to a greater difference between the ASAP and ALAP policies.

4. CONCLUDING COMMENTS

It should be emphasized that the DP model has considerably greater generality than was indicated in the limited application to Washington, D. C. The only model constraint on the data is that they be self-consistent (e.g., M_t and N_t must be consistent with D_t). If, for example, an urban fire department sees fit to reduce its fleet size because of overkill capacity or perhaps because of declining demand, and the values of M_t and N_t fluctuate because of a fluctuating budget, then a greater portion of the model's generality could be exploited. The interactions among the variables and parameters of the model which are evident in Appendix A should support this contention.

On the other hand, time limitations prevented any attempts to examine the model with particular relationships among the parameters. It seems reasonable that certain conditions, e.g., $M_t = N_t = \text{constant}$, or $D_t = \text{a constant for all } t$, could lead perhaps to closed-form optimal solutions, or at least might simplify the necessary DP calculations. Further research along these lines is recommended. In addition to these basic issues, there is a need for further sensitivity tests, with respect to the discount rate and the value of U_1 , for other values of the parameters M_t , N_t , D_t , and R . For instance, the optimal values of the objective $f_1(m, 0, 0)$ could be compared for different values of R (in some reasonable range of maximum ages), leading to an "optimal"

value of R (i.e. one which minimizes $f_1(m, 0, 0)$). Finally, runs with depreciation rate ρ varying, or using a different (perhaps linear) depreciation policy, would be desirable.

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APPENDIX A

DETAILS OF THE DYNAMIC PROGRAMMING MODEL

•

This Appendix develops certain details of the DP model described in Section 3. In particular, relationships among the variables are investigated which make it possible to examine a limited number of states and decisions for which the stage returns $I_t(x_t, d_t)$ are calculated.

Although technical in nature, this aspect of the problem is of great importance to computational feasibility in the sense that computer storage requirements and running times depend on the number of states and decisions the algorithm must consider.

The definitions of the relevant variables and parameters are repeated below for the reader's convenience:

x_{1t} = the number of engines remaining from the initial fleet
in year $t-1$ ($t=1, \dots, T+1$),

x_{2t} = the number of new engines purchased in years $1, \dots, t-1$
($t=1, \dots, T+1$),

x_{3t} = the maintenance cost during year $t-1$ on engines purchased
in years $1, \dots, t-1$ ($t=1, \dots, T+1$),

d_{1t} = the number of engines retired in year t ($t=1, \dots, T$),

d_{2t} = the number of engines purchased in year t ($t=1, \dots, T$),

D_t = the minimum number of engines required in year t ($t=1, \dots, T$),

M_t = the maximum number of engines which may be purchased in
year t ($t=1, \dots, T$),

N_t = the maximum number of engines which may be retired in
year t ($t=1, \dots, T$),

R = the age by which engines must be retired,

Q_a = the number of a-year-old engines in the initial fleet.

From these definitions, we may calculate two other quantities which are used throughout the sequel:

$$m = \sum_a Q_a = \text{the number of engines in the initial fleet,}$$

$$n_t = \sum_{a > R-t+1} Q_a = \text{the number of engines which must be retired}$$

prior to year t because of the age limitation $R(t=2, \dots, T+1)$.

Note that by definition: $x_{11} = m$, $x_{21} = 0$, $x_{31} = 0$, and $n_1 = 0$.

It is notationally convenient to adopt the convention $D_0 = m$.

Using the definitions above, we may immediately establish the relationships

$$x_{1t} = x_{1,t-1} - d_{1,t-1} \quad (t=2, \dots, T+1) \quad (\text{A-1})$$

$$x_{2t} = x_{2,t-1} + d_{2,t-1} \quad (t=2, \dots, T+1) \quad (\text{A-2})$$

$$0 \leq d_{1t} \leq N_t \quad (t=1, \dots, T) \quad (\text{A-3})$$

$$0 \leq d_{2t} \leq M_t \quad (t=1, \dots, T) \quad (\text{A-4})$$

$$x_{1t} + x_{2t} \geq D_{t-1} \quad (t = 1, \dots, T+1) \quad (\text{A-5})$$

$$\sum_{j=1}^{t-1} d_{1j} \geq n_t \quad (t=1, \dots, T+1) \quad (\text{A-6})$$

We maintain our convention regarding sums, viz., a sum is zero if its lower limit exceeds its upper limit. For example, (A-6) is valid for $t=1$ since both sides of the inequality are zero. The variations in the index-ranges are due to the fact that the state

variables refer to the system upon entering year t (or leaving year $t-1$), while the decision variables refer to decisions made in year t (presumed to be made at the beginning of year t).

Note that x_{3t} does not appear in (A-1) - (A-6). This is because x_{3t} depends only upon the distribution of the purchases x_{2t} over the years $1, \dots, t-1$. This observation is discussed at greater length subsequently.

It is clear that the stream of decisions $d_{2t} = M_t$ ($t=1, \dots, T$) and the resultant stream of states $x_{2t} = \sum_{j=1}^{t-1} M_j$ ($t=1, \dots, T$) do not violate (A-1) - (A-6).

Hence the least upper bound (LUB) of d_{2t} is

$$\mu(d_{2t}) = M_t \quad (t=1, \dots, T), \quad (\text{A-7})$$

and the LUB of x_{2t} is

$$\mu(x_{2t}) = \sum_{j=1}^{t-1} M_j \quad (t=2, \dots, T). \quad (\text{A-8})$$

[Recall that $x_{21} = 0$ by definition.] We use (A-7) and (A-8) to

develop the LUB and the greatest lower bound (GLB) of x_{1t}

($t=2, \dots, T+1$). [Recall that $x_{11} = m$ by definition.]

For a lower bound on x_{1t} , we observe first that for

$t \leq \tau \leq T+1$, $x_{1t} \geq x_{1\tau}$, so that

$$x_{1t} + \sum_{j=1}^{\tau-1} M_j \geq x_{1\tau} + \sum_{j=1}^{\tau-1} d_{2j} = x_{1\tau} + x_{2\tau} \geq D_{\tau-1},$$

implying that

$$x_{1t} \geq D_{\tau-1} - \sum_{j=1}^{\tau-1} M_j \quad (t \leq \tau \leq T+1). \quad (\text{A-9})$$

For $1 \leq \tau < t$, we have

$$x_{1\tau} = x_{1t} + \sum_{j=\tau}^{t-1} d_{1j},$$

so that

$$\begin{aligned} x_{1t} + \sum_{j=1}^{\tau-1} M_j &\geq x_{1\tau} - \sum_{j=\tau}^{t-1} d_{1j} + \sum_{j=1}^{\tau-1} d_{2j} \\ &\geq x_{1\tau} - \sum_{j=\tau}^{t-1} N_j + x_{2\tau} \\ &\geq D_{\tau-1} - \sum_{j=\tau}^{t-1} N_j, \end{aligned}$$

implying that

$$x_{1t} \geq D_{\tau-1} - \sum_{j=1}^{\tau-1} M_j - \sum_{j=\tau}^{t-1} N_j \quad (1 \leq \tau < t). \quad (\text{A-10})$$

With our convention concerning sums, (A-9) and (A-10) can be combined as

$$x_{1t} \geq \max_{1 \leq \tau \leq T+1} [D_{\tau-1} - \sum_{j=1}^{\tau-1} M_j - \sum_{j=\tau}^{t-1} N_j] \quad (t = 2, \dots, T+1),$$

where the case $\tau=1$ corresponds to the condition $x_{1t} \geq m - \sum_{j=1}^{t-1} N_j$.

We also require $x_{1t} \geq 0$. Hence

$$x_{1t} \geq \max \{0, \max_{1 \leq \tau \leq T+1} [D_{\tau-1} - \sum_{j=1}^{\tau-1} M_j - \sum_{j=\tau}^{t-1} N_j]\} \quad (t=2, \dots, T+1). \quad (A-11)$$

For an upper bound to x_{1t} , we note that for $t \leq \tau \leq T+1$,

$$n_{\tau} \leq \sum_{j=1}^{\tau-1} d_{1j} \leq \sum_{j=1}^{t-1} d_{1j} + \sum_{j=t}^{\tau-1} N_j = m - x_{1t} + \sum_{j=t}^{\tau-1} N_j, \text{ so that}$$

$$x_{1t} \leq m - \max_{t \leq \tau \leq T+1} [n_{\tau} - \sum_{j=t}^{\tau-1} N_j] \quad (t=2, \dots, T+1), \quad (A-12)$$

where the case $\tau=t$ corresponds to the condition $x_{1t} \leq m - n_t$.

We now let

$$\lambda(x_{1t}) = \max \{0, \max_{1 \leq \tau \leq T+1} [D_{\tau-1} - \sum_{j=1}^{\tau-1} M_j - \sum_{j=\tau}^{t-1} N_j]\} \quad (t=2, \dots, T+1), \quad (A-13)$$

$$\mu(x_{1t}) = m - \max_{t \leq \tau \leq T+1} [n_{\tau} - \sum_{j=t}^{\tau-1} N_j] \quad (t=2, \dots, T+1), \quad (A-14)$$

and we show that the formulas (A-13) and (A-14) give the GLB and

LUB of x_{1t} , respectively. This is accomplished by showing that

the $\lambda(x_{1t})$ ($t=1, \dots, T+1$) and $\mu(x_{1t})$ ($t=1, \dots, T+1$) are feasible

streams of the state variables x_{1t} . From (A-13) we know that

$$\lambda(x_{1t}) \geq D_{t-1} - \sum_{j=1}^{t-1} M_j, \text{ or}$$

$$\lambda(x_{1t}) + \sum_{j=1}^{t-1} M_j \geq D_{t-1}. \quad (\text{A-15})$$

Since $\lambda(x_{1t}) \leq \mu(x_{1t})$ must hold for the problem to be feasible, it follows that

$$\mu(x_{1t}) + \sum_{j=1}^{t-1} M_j \geq D_{t-1}. \quad (\text{A-16})$$

Next, (A-14) implies

$$\mu(x_{1t}) \leq m - n_t, \quad (\text{A-17})$$

from which it follows that

$$\lambda(x_{1t}) \leq m - n_t. \quad (\text{A-18})$$

Relations (A-14) and (A-18) imply, respectively, that demand is met in year $t-1$ with state $\lambda(x_{1t})$ and that required retirements

are met with state $\lambda(x_{1t})$. Relations (A-16) and (A-17) imply that these same two conditions are met by the state $\mu(x_{1t})$.

We now state an obvious fact.

Lemma 1. If $\{a_i\}$ and $\{b_i\}$ are finite sequences and k_1 and k_2 are constants such that $k_1 \leq a_i - b_i \leq k_2$ for all i , then $k_1 \leq \max a_i - \max b_i \leq k_2$.

In order to show that $\lambda(x_{1t})$ and $\mu(x_{1t})$ are feasible streams, it remains only to show the following two propositions.

Proposition 1. $0 \leq \lambda(x_{1t}) - \lambda(x_{1,t+1}) \leq N_t$ ($t=1, \dots, T+1$).

Proof. For arbitrary t , let $a_0 = b_0 = 0$, and let

$$a_\tau = D_{\tau-1} - \sum_{j=1}^{\tau-1} M_j - \sum_{j=\tau}^{t-1} N_j \quad (\tau=1, \dots, T+1),$$

$$b_\tau = D_{\tau-1} - \sum_{j=1}^{\tau-1} M_j - \sum_{j=\tau}^t N_j \quad (\tau=1, \dots, T+1).$$

It is clear that $0 \leq a_\tau - b_\tau \leq N_t$ ($\tau=0, \dots, T+1$), so that Lemma 1

implies $0 \leq \max_{0 \leq \tau \leq T+1} a_\tau - \max_{0 \leq \tau \leq T+1} b_\tau \leq N_t$, or $0 \leq \lambda(x_{1t}) - \lambda(x_{1,t+1}) \leq N_t$,

as stated.

Proposition 2. $0 \leq \mu(x_{1t}) - \mu(x_{1,t+1}) \leq N_t \quad (t=1, \dots, T+1).$

Proof. For arbitrary t , let $a_{t+1} = n_{t+1}$, $b_{t+1} = \max [n_t, n_{t+1} - N_t]$,

$$a_\tau = n_\tau - \sum_{j=t+1}^{\tau-1} N_j \quad (\tau=t+2, \dots, T+1),$$

$$b_\tau = n_\tau - \sum_{j=t+1}^{\tau-1} N_t \quad (\tau=t+2, \dots, T+1).$$

For $\tau=t+2, \dots, T+1$, it is clear that $0 \leq a_\tau - b_\tau \leq N_t$. If $b_{t+1} = n_t$,

then $a_{t+1} - b_{t+1} = n_{t+1} - n_t \geq 0$ by definition, and in this case

$n_t \geq n_{t+1} - N_t$, so that $a_{t+1} - b_{t+1} = n_{t+1} - n_t \leq N_t$. If

$b_{t+1} = n_{t+1} - N_t$, then $a_{t+1} - b_{t+1} = n_{t+1} - (n_{t+1} - N_t) = N_t \geq 0$.

Hence $0 \leq a_\tau - b_\tau \leq N_t \quad (\tau=t+1, \dots, T+1)$, so that Lemma 1 implies

$0 \leq \max_\tau a_\tau - \max_\tau b_\tau \leq N_t$. Therefore ,

$$0 \leq (m - \max_\tau b_\tau) - (m - \max_\tau a_\tau) = \mu(x_{1t}) - \mu(x_{1,t+1}) \leq N_t.$$

Propositions 1 and 2 imply that $\lambda(x_{1,t+1})$ can be "reached" from $\lambda(x_{1t})$ with a feasible decision, and that $\mu(x_{1,t+1})$ can be "reached" from $\mu(x_{1t})$ with a feasible decision, respectively. [See (A-3).]

These propositions together with the conclusions drawn from (A-15) - (A-18) imply that the $\lambda(x_{1t})$ and the $\mu(x_{1t})$ are feasible streams, and this together with (A-11) and (A-12) in turn imply that $\lambda(x_{1t})$ and $\mu(x_{1t})$ are the GLB and the LUB of x_{1t} , respectively.

Fix a value of x_{1t} , say \hat{x}_{1t} , with $\lambda(x_{1t}) \leq \hat{x}_{1t} \leq \mu(x_{1t})$. We now develop the GLB of x_{2t} , given \hat{x}_{1t} . For $t < \tau \leq T+1$, we have

$$x_{1\tau} \leq \mu(x_{1\tau}) \text{ and } x_{1\tau} \leq \hat{x}_{1t}, \text{ and so } x_{1\tau} \leq \min [\hat{x}_{1t}, \mu(x_{1\tau})].$$

Hence,

$$\min[\hat{x}_{1t}, \mu(x_{1\tau})] + x_{2t} + \sum_{j=t}^{\tau-1} M_j \geq x_{1\tau} + x_{2\tau} \geq D_{\tau-1},$$

so

$$x_{2t} \geq \max_{t < \tau \leq T+1} \{D_{\tau-1} - \sum_{j=t}^{\tau-1} M_j - \min [\hat{x}_{1t}, \mu(x_{1\tau})]\}. \quad (\text{A-19})$$

$$\text{For } 1 \leq \tau \leq t, x_{1\tau} \leq \mu(x_{1\tau}) \text{ and } x_{1\tau} = \hat{x}_{1t} + \sum_{j=\tau}^{t-1} d_{1j} \leq \hat{x}_{1t} + \sum_{j=\tau}^{t-1} N_j.$$

Thus $x_{1\tau} \leq \min[\mu(x_{1\tau}), \hat{x}_{1t} + \sum_{j=\tau}^{t-1} N_j] \triangleq x_{1\tau}^*$. [The notation " \triangleq " means

"defined as."] Essentially $x_{1\tau}^*$ is the largest value of $x_{1\tau}$ such

that \hat{x}_{1t} can be "reached" from it by feasible decisions.

Now, for $\tau \leq \sigma \leq t$

$$x_{1\sigma}^* + x_{2\tau} + \sum_{j=\tau}^{\sigma-1} M_j \geq x_{1\sigma}^* + x_{2\sigma} \geq D_{\sigma-1}.$$

$$\text{Hence } x_{2\tau} \geq D_{\sigma-1} - x_{1\sigma}^* - \sum_{j=\tau}^{\sigma-1} M_j. \quad \text{For } 1 \leq \sigma \leq \tau,$$

$$x_{1\sigma}^* + x_{2\tau} \geq x_{1\sigma}^* + x_{2\sigma} \geq D_{\sigma-1},$$

implying $x_{2\tau} \geq D_{\sigma-1} - x_{1\sigma}^*$. Again, using our convention regarding

sums, we have

$$x_{2\tau} \geq \max_{1 \leq \sigma \leq t} [D_{\sigma-1} - x_{1\sigma}^* - \sum_{j=\tau}^{\sigma-1} M_j] \triangleq x_{2\tau}^* \quad (1 \leq \tau \leq t), \quad (\text{A-20})$$

and in particular

$$x_{2t} \geq \max_{1 \leq \tau \leq t} [D_{\sigma-1} - x_{1\sigma}^*] = \max_{1 \leq \tau \leq t} \{D_{\tau-1} - \min[\mu(x_{1\tau}), \hat{x}_{1t} + \sum_{j=\tau}^{t-1} N_j]\}. \quad (\text{A-21})$$

Let

$$\lambda(x_{2t}; \hat{x}_{1t}) = \max_{1 \leq \tau \leq T+1} \{D_{\tau-1} - \sum_{j=t}^{\tau-1} M_j - \min[\mu(x_{1\tau}), \hat{x}_{1t} + \sum_{j=\tau}^{t-1} N_j]\}. \quad (\text{A-22})$$

For $1 \leq \tau \leq t$ (A-22) reduces to (A-21), and for $t < \tau \leq T+1$ (A-22) reduces to

(A-19), so that $\lambda(x_{2t}; \hat{x}_{1t})$ is a lower bound on x_{2t} , given \hat{x}_{1t} . We show

that $\lambda(x_{2t}; \hat{x}_{1t})$ is the GLB of x_{2t} , given \hat{x}_{1t} , by first showing the

existence of a feasible stream to $\lambda(x_{2t}; \hat{x}_{1t})$ and then showing

that this state can be completed into the future with a stream

feasible in years τ for $t < \tau \leq T+1$. It is easily shown that the

feasibility condition $\lambda(x_{2t}; \lambda(x_{1t})) \leq \mu(x_{2t})$ follows from

the feasibility condition $\lambda(x_{1t}) \leq \mu(x_{1t})$.

First, note that $(x_{1t}^*, x_{2t}^*) = (\hat{x}_{1t}, \lambda(x_{2t}; \hat{x}_{1t}))$. We show that the sequence $\{(x_{1\tau}^*, x_{2\tau}^*)\}$ ($\tau = 1, \dots, t$) is the desired stream.

That $x_{1\tau}^* \leq \mu(x_{1\tau})$ follows from the definition of $x_{1\tau}^*$. Since $\mu(x_{1\tau}) \geq \lambda(x_{1\tau})$ and

$$\hat{x}_{1t} + \sum_{j=\tau}^{t-1} N_j \geq \lambda(x_{1t}) + \sum_{j=\tau}^{t-1} [\lambda(x_{1j}) - \lambda(x_{1,j+1})] = \lambda(x_{1\tau}),$$

we have $x_{1\tau}^* \geq \lambda(x_{1\tau})$. (The inequality in the above expression follows from $\hat{x}_{1t} \geq \lambda(x_{1t})$ and Proposition 1.) Therefore, $\lambda(x_{1\tau}) \leq x_{1\tau}^* \leq \mu(x_{1\tau})$.

Next we observe that $0 \leq \mu(x_{1\tau}) - \mu(x_{1,\tau+1}) \leq N_t$, by Proposition 2

and that $\hat{x}_{1t} + \sum_{j=\tau}^{t-1} N_j - (\hat{x}_{1t} + \sum_{j=\tau+1}^{t-1} N_j) = N_\tau$. It

follows for all combinations of cases for $x_{1\tau}^*$ and $x_{1,\tau+1}^*$, that

$0 \leq x_{1\tau}^* - x_{1,\tau+1}^* \leq N_\tau$. We have already seen that $x_{2\tau}^* \geq D_{\tau-1} - x_{1\tau}^*$,

so that $x_{1\tau}^* + x_{2\tau}^* \geq D_{\tau-1}$. Thus far we have shown that the sequence

$\{x_{1\tau}^*\}$ ($\tau=1, \dots, t$) is feasible. To complete the proof for

$x_{2\tau}^*$, it remains to show that $0 \leq x_{2,\tau+1}^* - x_{2\tau}^* \leq M_\tau$. This result

follows from Lemma 1 with $a_\sigma = D_{\sigma-1} - x_{1\sigma}^* - \sum_{j=\tau+1}^{\sigma-1} M_j$,

$b_\sigma = D_{\sigma-1} - x_{1\sigma}^* - \sum_{j=\tau}^{\sigma-1} M_j$, since $a_\sigma - b_\sigma = M_\tau$ for $\tau+1 \leq \sigma \leq t$ and

$a_\sigma - b_\sigma = 0$ for $1 \leq \sigma \leq \tau$.

To show that $(\hat{x}_{1t}, \lambda(x_{2t}; \hat{x}_{1t}))$ can be completed into the future to a stream feasible in years $t < \tau \leq T+1$, we choose $d_{2\tau} = M_\tau$ and choose $d_{1\tau}$ so that $x_{1\tau} = \min[\hat{x}_{1t}, \mu(x_{1\tau})]$. Note that the sequence $\{x_{1\tau}\}$ ($\tau = t+1, \dots, T+1$) is non-increasing. That the condition $x_{1\tau} + x_{2\tau} \geq D_{\tau-1}$ holds is a direct consequence of the way $\lambda(x_{2t}; \hat{x}_{1t})$ was derived. Next, $x_{1\tau} \leq \mu(x_{1\tau}) \leq m - n_\tau$, so that the required number of retirements is met. We need only show that $0 \leq x_{1\tau} - x_{1,\tau+1} \leq N_\tau$ ($t < \tau \leq T$) to complete the proof. The left-hand inequality is clear from Proposition 2. For the right-hand inequality, if $x_{1\tau} = \mu(x_{1\tau})$, then $\mu(x_{1,\tau+1}) \leq \mu(x_{1\tau}) \leq \hat{x}_{1t}$, so that $x_{1\tau} - x_{1,\tau+1} \leq \mu(x_{1\tau}) - \mu(x_{1,\tau+1}) \leq N_t$ by Proposition 2. If $x_{1\tau} = x_{1,\tau+1} = \hat{x}_{1t}$, then the result is clear. If $x_{1\tau} = \hat{x}_{1t}$ and $x_{1,\tau+1} = \mu(x_{1,\tau+1})$, then $x_{1\tau} - x_{1,\tau+1} = \hat{x}_{1t} - \mu(x_{1,\tau+1}) \leq \mu(x_{1\tau}) - \mu(x_{1,\tau+1}) \leq N_t$, again by Proposition 2.

We now assume given a state \hat{x}_{1t} , $\lambda(x_{1t}) \leq \hat{x}_{1t} \leq \mu(x_{1t})$, and a state \hat{x}_{2t} , $\lambda(x_{2t}; \hat{x}_{1t}) \leq \hat{x}_{2t} \leq \mu(x_{2t})$, and we derive bounds on x_{3t} ($2 \leq t \leq T$). [Recall that $x_{31} = 0$ by definition.] For the given states $\hat{x}_{1t}, \hat{x}_{2t}$ these bounds $\lambda(x_{3t}; \hat{x}_{1t}, \hat{x}_{2t})$ and $\mu(x_{3t}; \hat{x}_{1t}, \hat{x}_{2t})$ correspond to a "purchase late" scenario and a "purchase early" scenario, respectively, i.e. the smallest value of x_{3t} is realized when the \hat{x}_{2t} engines are purchased as close to t as possible,

while the largest value of x_{3t} is realized when the \hat{x}_{2t} engines are purchased as distant from t as possible. These intuitive concepts are formulated mathematically in the following paragraphs.

For the "purchase late" scenario, we observe that

$\hat{x}_{2t} - \sum_{j=\tau}^{t-1} M_j$ is the smallest value of $x_{2\tau}$ which can "reach"

\hat{x}_{2t} . Hence $x_{2\tau} \geq \hat{x}_{1t} - \sum_{j=\tau}^{t-1} M_j$. We also have $x_{2\tau} \geq x_{2\tau}^*$, as

derived above. Combining these we have

$$x_{2\tau} \geq \max[x_{2\tau}^*, \hat{x}_{1t} - \sum_{j=\tau}^{t-1} M_j] \triangleq \bar{x}_{2\tau} \quad (2 \leq \tau \leq t).$$

To show that the $\bar{x}_{2\tau}$ correspond to the GLB of x_{3t} , given \hat{x}_{1t} and \hat{x}_{2t} , it suffices to show that the sequence $\{(x_{1\tau}^*, \bar{x}_{2\tau})\}$ ($\tau=1, \dots, t$) is feasible. We have already shown that $x_{1\tau}^*$ is in the appropriate

range and that $0 \leq x_{1t}^* - x_{1,\tau+1}^* \leq N_\tau$. That demand is met follows

from $x_{1\tau}^* + \bar{x}_{2\tau} \geq x_{1\tau}^* + x_{2\tau}^* \geq D_{\tau-1}$. Finally, $0 \leq \bar{x}_{2,\tau+1} - \bar{x}_{2\tau} \leq M_\tau$

follows from $0 \leq x_{2,\tau+1}^* - x_{2\tau}^* \leq M_\tau$ and $\hat{x}_{2t} - \sum_{j=\tau+1}^{t-1} M_j - (\hat{x}_{2t} - \sum_{j=\tau}^{t-1} M_j) = M_\tau$,

for all combinations of cases for $\bar{x}_{2\tau}$ and $\bar{x}_{2,\tau+1}$. The number of

purchases made in year τ in the "purchase late" scenario is

$\bar{x}_{2,\tau+1} - \bar{x}_{2\tau}$, so that

$$\lambda(x_{3t}; \hat{x}_{1t}, \hat{x}_{2t}) = \sum_{\tau=1}^{t-1} u_{t-\tau} (\bar{x}_{2,\tau+1} - \bar{x}_{2\tau}) \quad (t=2, \dots, T). \quad (A-23)$$

The "purchase early" scenario is somewhat simpler. We have

$$\hat{x}_{2t} \leq \sum_{j=1}^{t-1} M_j, \text{ so there exists a largest } \tau (2 \leq \tau \leq t), \text{ say } \tau = \sigma,$$

$$\text{for which } \hat{x}_{2t} \geq \sum_{j=1}^{\sigma-1} M_j. \text{ Let } \tilde{x}_{2\tau} = \sum_{j=1}^{\tau-1} M_j \text{ for } 1 \leq \tau < \sigma \text{ and } \tilde{x}_{2\tau} = \hat{x}_{2t}$$

for $\sigma \leq \tau \leq t$. The sequence $\{\tilde{x}_{2\tau}\}$ ($\tau=1, \dots, t-1$) is clearly feasible.

Hence

$$\mu(x_{3t}; \hat{x}_{1t}, \hat{x}_{2t}) = \sum_{\tau=1}^{t-1} u_{t-\tau} (\tilde{x}_{2,\tau+1} - \tilde{x}_{2\tau}) \quad (t=2, \dots, T). \quad (A-24)$$

Note that \hat{x}_{1t} does not appear explicitly in this derivation.

It was stated in Section 3 that the linear form of the maintenance cost function yields a desirable property of the range of x_{3t} , viz.,

that its values are precisely multiples of the slope U_1 of the linear

function. With $u_a = U_0 + U_1 a$, $y_\tau = \bar{x}_{2,\tau+1} - \bar{x}_{2\tau}$ is the number of purchases

in year τ corresponding to the "purchase late" scenario.

Let z_τ be any feasible number of purchases in year τ , given \hat{x}_{1t}

and \hat{x}_{2t} . Then

$$x_{3t} - \lambda(x_{3t}; \hat{x}_{1t}, \hat{x}_{2t}) = \sum_{\tau=1}^{t-1} u_{t-\tau} z_\tau - \sum_{\tau=1}^{t-1} u_{t-\tau} y_\tau$$

$$= \sum_{\tau=1}^{t-1} [U_0 + U_1(t-\tau)] (z_{\tau} - y_{\tau}) = U_1 \left[\sum_{\tau=1}^{t-1} \tau (z_{\tau} - y_{\tau}) \right],$$

where the last equality follows from the fact that

$$\sum_{\tau=1}^{t-1} y_{\tau} = \sum_{\tau=1}^{t-1} z_{\tau} = \hat{x}_{2t}.$$

Note that the number of values for x_{3t} is $\sum_{\tau=1}^{t-1} \tau (z_{\tau} - y_{\tau}) + 1$.

We turn now to establishing bounds on the decision variables.

Assume fixed values of x_{1t} and x_{2t} in their appropriate ranges, say

$\lambda(x_{1t}) \leq \hat{x}_{1t} \leq \mu(x_{1t})$ and $\lambda(x_{2t}; \hat{x}_{1t}) \leq \hat{x}_{2t} \leq \mu(x_{2t})$; the state variable

x_3 does not play a role. We know that $d_{1t} \geq 0$ from (A-3), and that

$x_{1t} - d_{1t} \leq \mu(x_{1,t+1})$. Thus

$$\lambda(d_{1t}; \hat{x}_{1t}) = \max [0, \hat{x}_{1t} - \mu(x_{1,t+1})] \quad (t=1, \dots, T). \quad (\text{A-25})$$

(Note that $\lambda(d_{1t}; \hat{x}_{1t})$ is not a function of \hat{x}_{2t} .)

Relations (A-3) also state that $d_{1t} \leq N_t$, and we have

$\hat{x}_{1t} - d_{1t} \geq \lambda(x_{1,t+1})$. In addition to these constraints, d_{1t} must be

chosen so that the resulting $x_{1,t+1}$ yields a lower bound on $x_{2,t+1}$ that

can be "reached" from \hat{x}_{2t} , i.e. $\hat{x}_{2t} + M_t \geq \lambda(x_{2,t+1}; \hat{x}_{1t} - d_{1t})$.

Using the definition of $\lambda(x_{2,t+1}; \hat{x}_{1t} - d_{1t})$, we have

$$\begin{aligned}
\hat{x}_{2t} + M_t &\geq \max_{1 \leq \tau \leq t+1} \{ D_{\tau-1} - \sum_{j=\tau}^{\tau-1} M_j - \min [\mu(x_{1\tau}), \hat{x}_{1t} - d_{1t} + \sum_{j=\tau}^t N_j] \} \\
&= \max \{ \max_{1 \leq \tau \leq T+1} [D_{\tau-1} - \sum_{j=\tau}^{\tau-1} M_j - \mu(x_{1\tau})], \\
&\quad \max_{1 \leq \tau \leq T+1} [D_{\tau-1} - \sum_{j=\tau}^{\tau-1} M_j - \hat{x}_{1t} + d_{1t} - \sum_{j=\tau}^t N_j] \}.
\end{aligned} \tag{A-26}$$

The part of (A-26) involving d_{1t} becomes

$$d_{1t} \leq \hat{x}_{1t} + \hat{x}_{2t} + M_t - \max_{1 \leq \tau \leq T+1} [D_{\tau-1} - \sum_{j=\tau}^{\tau-1} M_j - \sum_{j=\tau}^t N_j].$$

Therefore, we take

$$\begin{aligned}
\mu(d_{1t}; \hat{x}_{1t}, \hat{x}_{2t}) &= \min \{ N_t, \hat{x}_{1t} - \lambda(x_{1,t+1}), \\
&\quad \hat{x}_{1t} + \hat{x}_{2t} + M_t - \max_{1 \leq \tau \leq T+1} [D_{\tau-1} - \sum_{j=\tau}^{\tau-1} M_j - \sum_{j=\tau}^t N_j] \}.
\end{aligned} \tag{A-27}$$

That $\lambda(d_{1t}; \hat{x}_{1t}, \hat{x}_{2t}) \leq \mu(d_{1t}; \hat{x}_{1t}, \hat{x}_{2t})$ holds may be shown straightforwardly

by taking $\hat{x}_{2t} = \lambda(x_{2t}; \hat{x}_{1t})$ in the μ term and applying the definitions

in (A-25) and (A-27).

Since we already have $\mu(d_{2t}) = M_t$ from (A-4), it remains only to find

$\lambda(d_{2t}; \hat{x}_{1t}, \hat{x}_{2t}, \hat{d}_{1t})$, where the three given variables fall in their

respective ranges. Relations (A-4) state that $d_{2t} \geq 0$. In addition,

we require $\hat{x}_{2t} + d_{2t} \geq \lambda(x_{2,t+1}; \hat{x}_{1t} - \hat{d}_{1t})$, so that

$$\lambda(d_{2t}; \hat{x}_{1t}, \hat{x}_{2t}, \hat{d}_{1t}) = \max [0, \lambda(x_{2,t+1}; \hat{x}_{1t} - \hat{d}_{1t}) - \hat{x}_{2t}] \quad (t=1, \dots, T). \tag{A-28}$$

That $\lambda(d_{2t}; \hat{x}_{1t}, \hat{x}_{2t}, \hat{d}_{1t}) \leq \mu(d_{2t}) = M_t$ holds also is straightforward to verify.

Observe that the ranges of d_{1t} and d_{2t} , developed above, do not depend on x_{3t} . In fact, we show below that the optimal decisions at any stage are independent of x_{3t} , because given \hat{x}_{1t} and \hat{x}_{2t} , the value of the objective $f_t(x_t)$ at each stage is a linear function of x_{3t} , with the specific form

$$f_t(x_t) = g_t(x_{1t}, x_{2t}) + \left(\sum_{j=0}^{T-t} \delta^j \right) x_{3t}$$

where $\delta = 1/(1+r)$. For $t=T$, equations (3.3) and (3.4) imply

$$\begin{aligned} f_T(x_T) &= \min_{d_T} I_T(x_T, d_T) \\ &= g_T(x_{1T}, x_{2T}) + x_{3T}, \end{aligned}$$

with g_T taken as that part of (3.3) not involving x_{3T} . Now assuming

that $f_t(x_t) = h_t(x_{1t}, x_{2t}) + \left(\sum_{j=0}^{T-t} \delta^j \right) x_{3t}$, we show that

$f_{t-1}(x_{t-1}) = h_{t-1}(x_{1,t-1}, x_{2,t-1}) + \left(\sum_{j=0}^{T-t+1} \delta^j \right) x_{3,t-1}$, (i.e., "backwards induction" on t). We have

$$\begin{aligned} f_{t-1}(x_{t-1}) &= \min_{d_{t-1}} [I_{t-1}(x_{t-1}, d_{t-1}) + \delta f_t(x_t)] \\ &= \min_{d_{t-1}} [I_{t-1} + \delta h_t + \delta \left(\sum_{j=0}^{T-t} \delta^j \right) (x_{3,t-1} + u_1 d_{2,t-1} + U_1 x_{2,t-1})] \end{aligned}$$

$$= \min_{d_{t-1}} [g_{t-1} + \delta h_t + (\sum_{j=1}^{T-t+1} \delta^j) (u_1 d_{2,t-1} + u_1 x_{2,t-1}) + (\sum_{j=1}^{T-t+1} \delta^j) x_{3,t-1} + x_{3,t-1}],$$

where g_{t-1} is that part of I_{t-1} not involving $x_{3,t-1}$ (cf. equation ()).

$$f_{t-1}(x_{t-1}) = \min_{d_{t-1}} [h'_{t-1}(x_{1,t-1}, x_{2,t-1}) + (\sum_{j=0}^{T-t+1} \delta^j) x_{3,t-1}]$$

$$= h_{t-1}(x_{1,t-1}, x_{2,t-1}) + (\sum_{j=0}^{T-t+1} \delta^j) x_{3,t-1}$$

$$\text{where } h'_{t-1} = g_{t-1} + \delta h_t + (\sum_{j=1}^{T-t+1} \delta^j) (u_1 d_{2,t-1} + u_1 x_{2,t-1}).$$

The fact just proven makes it unnecessary to cycle through all of the values of d_{1t} and d_{2t} for each x_{3t} . We need only determine the optimal decisions for one value of x_{3t} , say $\lambda(x_{3t}; \hat{x}_{1t}, \hat{x}_{2t})$; these decisions are optimal for other values of x_{3t} , given \hat{x}_{1t} and \hat{x}_{2t} , and the corresponding values of $f_t(x_t)$ may be calculated simply by adding the appropriate multiple of

$$(\sum_{j=0}^{T-t} \delta^j) \text{ to the optimal value of the objective function for } \lambda(x_{3t}; \hat{x}_{1t}; \hat{x}_{2t}).$$

Table A-1 gives a summary of all formulas needed to calculate the ranges of the variables used in the dynamic programming model.

Table A-1 - FORMULAS FOR RANGES OF THE DYNAMIC PROGRAMMING MODEL VARIABLES

$$\lambda(x_{1t}) = \max \{0, \max_{1 \leq \tau \leq T+1} [D_{\tau-1} - \sum_{j=1}^{\tau-1} M_j - \sum_{j=\tau}^{t-1} N_j]\}$$

$$\mu(x_{1t}) = m - \max_{t \leq \tau \leq T+1} [n_{\tau} - \sum_{j=t}^{\tau-1} N_j]$$

$$\lambda(x_{2t}; \hat{x}_{1t}) = \max_{1 \leq \tau \leq T+1} \{D_{\tau-1} - \sum_{j=\tau}^{\tau-1} M_j - \min [\mu(x_{1\tau}), \hat{x}_{1t} + \sum_{j=\tau}^{t-1} N_j]\}$$

$$\mu(x_{2t}) = \sum_{j=1}^{t-1} M_j$$

$$\lambda(x_{3t}; \hat{x}_{1t}, \hat{x}_{2t}) = \sum_{\tau=1}^{t-1} U_{t-\tau} (\bar{x}_{2,\tau+1} - \bar{x}_{2\tau})$$

$$\bar{x}_{2\tau} = \max [x_{2\tau}^*, \hat{x}_{1t} - \sum_{j=\tau}^{t-1} M_j]$$

$$x_{2\tau}^* = \max_{1 \leq \sigma \leq t} [D_{\sigma-1} - x_{1\sigma}^* - \sum_{j=\tau}^{\sigma-1} M_j]$$

$$x_{1\sigma}^* = \min [\mu(x_{1\sigma}), \hat{x}_{1t} + \sum_{j=\sigma}^{t-1} N_j]$$

$$\mu(x_{3t}; \hat{x}_{1t}, \hat{x}_{2t}) = \sum_{\tau=1}^{t-1} U_{t-\tau} (\tilde{x}_{2,\tau+1} - \tilde{x}_{2\tau}).$$

$$\tilde{x}_{2\tau} = \sum_{j=1}^{\tau-1} M_j \text{ for } 1 \leq \tau < \sigma$$

$$\tilde{x}_{2\tau} = \hat{x}_{2t} \text{ for } \sigma \leq \tau \leq t.$$

σ is the largest value of k such that $\hat{x}_{2t} > \sum_{j=1}^{k-1} M_j$.

$$\lambda(d_{1t}; \hat{x}_{1t}) = \max [0, \hat{x}_{1t} - \mu(x_{1,t+1})]$$

$$\mu(d_{1t}; \hat{x}_{1t}, \hat{x}_{2t}) = \min \{N_t, \hat{x}_{1t} - \lambda(x_{1,t+1}), x_{1t} + x_{2t} + M_t$$

$$- \max_{1 \leq \tau \leq T+1} [D_{\tau-1} - \sum_{j=t+1}^{\tau-1} M_j - \sum_{j=\tau}^t N_j]\}$$

$$\lambda(d_{2t}; \hat{x}_{1t}, \hat{x}_{2t}, \hat{d}_{1t}) = \max [0, \lambda(x_{2,t+1}; \hat{x}_{1t} - \hat{d}_{1t}) - \hat{x}_{2t}].$$

$$\mu(d_{2t}) = M_t.$$

APPENDIX B

LISTING OF THE COMPUTER CODE FOR THE DYNAMIC

PROGRAMMING MODEL


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00100 1* REPLAC IS A DYNAMIC PROGRAMMING CODE DESIGNED FOR AN EQUIPMENT
00100 2* REPLACEMENT PROBLEM
00100 3*
00100 4*
00100 5* IMPLICIT INTEGER (A-H,O-Z)
00100 6* REAL RATE,DELTA,DELTB,DELTAM
00100 7*
00100 8* NYRS IS THE MAXIMUM VALUE OF TT, I.E. THE MAXIMUM NUMBER OF YEARS
00100 9* IN THE PLANNING HORIZON OR THE MAXIMUM NUMBER OF STAGES.
00100 10* NPCS IS THE MAXIMUM VALUE OF MV, I.E. THE MAXIMUM NUMBER OF
00100 11* RESOURCES IN THE INITIAL FLEET.
00100 12* MAXAGE IS THE MAXIMUM VALUE OF R.
00100 13* NPOS IS THE MAXIMUM NUMBER OF VALUES FOR X1T IN ANY STAGE T.
00100 14* MPOS IS THE MAXIMUM NUMBER OF VALUES FOR X2T IN ANY STAGE T, I.E.
00100 15* THE NUMBER OF VALUES OF X2T ASSOCIATED WITH A SINGLE VALUE OF X1T
00100 16* SUMMED OVER ALL VALUES OF X1T FOR STAGE T.
00100 17* STATE IS THE MAXIMUM NUMBER OF STATES PER STAGE.
00100 18* NOX1 IS THE TOTAL NUMBER OF VALUES OF X1T IN ALL YEARS T, I.E. THE
00100 19* SUM OF THE NUMBER OF VALUES OF X1T IN YEAR 1, THE NUMBER OF VALUES
00100 20* IN YEAR 2, ETC.
00100 21*
00100 22* PARAMETER NYRS=25,NPCS=100,MAXAGE=25,STATE=10000
00100 23* PARAMETER NPOS=NPCS/4,MPOS=NPOS**2,NYRS1=NYRS+1,STATE2=2*STATE
00100 24* PARAMETER NOX1=NYRS1*NPCS
00100 25* COMMON M(NYRS),P(NYRS),LX1(NYRS1),MX1(NYRS1),LX2X1(NOX1),RN,DELTA,
00100 26* MX2(NYRS1),FTP1(STATE),IS(NPCS),D(NYRS),JNDX1(NPOS),U0,U1,
00100 27* JNDX2(MPOS),FLAG,RTPI,INDEX,V(MAXAGE,NYRS),NX1(NYRS1),TT,
00100 28* NN(NYRS),Q(MAXAGE),N(NYRS1),FT(STATE),INDX1(NPOS),
00100 29* INDX2(MPOS),DN(STATE2),VM,NOX11
00100 30* COMMON X1T,X2T,X3T,D1T,D2T,Y1TP1,Y2TP1,Y3TP1,T
00100 31* NOX11 = NOX1
00100 32* IOUT = 39
00100 33*
00100 34* IT IS THE NUMBER OF YEARS IN THE PLANNING HORIZON (THE NUMBER OF
00100 35* STAGES)
00100 36* VM IS THE NUMBER OF RESOURCES INITIALLY ON HAND
00100 37* R IS THE MAXIMUM ALLOWABLE AGE OF RESOURCES (RESOURCES OF AGE R
00100 38* INITIALLY MUST BE RETIRED IN YEAR 1)
00100 39*
00100 40* READ (5,800) TT,MV,R
00100 41* 800 FORMAT (3I5)
00100 42* J0 AND U1 ARE COEFFICIENTS OF THE LINEAR MAINTENANCE FUNCTION.
00100 43* MAINTENANCE COSTS ON A RESOURCE OF AGE A IS CALCULATED AS

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44*      J0 + J1*A.  MAINTENANCE COSTS ARE IN PENNIES.
45*      BASE IS THE YEAR AROUND WHICH PURCHASE PRICES ARE BASED.
46*      PU AND P1 ARE COEFFICIENTS OF THE PURCHASE PRICE FUNCTION.
47*      DEPRECIATION IS CALCULATED AS A GEOMETRIC DECREASE IN PURCHASE
48*      PRICE OVER R YEARS.  THE RATE OF DEPRECIATION IS (1 - RATE).
49*      PURCHASE PRICES ARE IN PENNIES.
50*      READ (5,805) J0,J1,BASE,P0,P1
51*      805
52*      FORMAT (5I10)
53*
54*      N(T) IS THE MAXIMUM NUMBER OF PURCHASES ALLOWED IN YEAR T
55*
56*      READ (5,810) (N(T),T=1,TT)
57*      810
58*      FORMAT (10I5)
59*
60*      NN(T) IS THE MAXIMUM NUMBER OF RETIREMENTS ALLOWED IN YEAR T
61*
62*      READ (5,810) (NN(T),T=1,TT)
63*
64*      Q(T) IS THE NUMBER OF RESOURCES OF AGE I IN THE INITIAL FLEET
65*
66*      READ (5,810) (Q(I),I=1,R)
67*
68*      D(T) IS THE MINIMUM NUMBER OF RESOURCES REQUIRED IN YEAR T
69*
70*      READ (5,810) (D(T),T=1,TT)
71*      BASE = BASE-1901
72*      RATE = .6
73*      DELTA = .085
74*
75*      V(A,T) IS THE SALVAGE VALUE IN YEAR T OF A RESOURCE WHICH WAS
76*      INITIALLY OF AGE A
77*
78*      DO 2 A=1,R
79*      C=P0+P1*(BASE-A+1)
80*      DO 2 I=1,TT
81*      V(A,T) = 0
82*      IF (A+T-1 .LE. R) V(A,T) = FLOAT(C)*RATE**(A+T-1) + .5
83*      2 CONTINUE
84*      WRITE (6,989) ((V(A,T),T=1,TT),A=1,R)
85*      989
86*      FORMAT (5I10)
87*      J=0
88*
89*      IS(J) IS THE AGE OF THE J-TH RESOURCE IN THE INITIAL FLEET.
90*      RESOURCE 1 IS YOUNGEST.
91*
92*      DO 5 I=1,R
93*      K=Q(I)
94*      IF (K .EQ. 0) GO TO 5
95*      DO 4 I1=1,K
96*      J=J+1
97*      IS(J) = I
98*      4 CONTINUE
99*      5 CONTINUE
100*      TTPI = TT+1
101*      YEAR T
102*      N(T) IS THE NUMBER OF RESOURCES WHICH MUST BE RETIRED PRIOR TO
103*      YEAR T

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00234 102* DO 30 T=1,TTPI
00235 103* IF (T .LT. TTPI) P(T) = P0+P1*(BASE+T)
00240 104* IF (T .GT. 1) GO TO 10
00242 105* N(1) = 0
00244 106* GO TO 30
00245 107*
00246 108* 10 M(T) = N(1-1)+Q(R-T+2)
00247 109* 30 CONTINUE
00247 110*
00247 111* C COMPUTE LX1(T) AND MX1(T), THE LOWER AND UPPER LIMITS FOR X1T.
00247 112* C X1T IS THE NUMBER OF RESOURCES REMAINING FROM THE INITIAL FLEET
00247 113* C IN YEAR T-1.
00247 114* C
00251 115* DO 70 T=1,TTPI
00254 116* DO 60 TAU=1,TTPI
00257 117* IF (TAU .GT. 1) GO TO 35
00261 118* SUM = WM
00262 119* GO TO 45
00263 120* 35 TAJM1 = TAU-1
00264 121* SUM = D(TAJM1)
00265 122* DO 40 J=1,TAJM1
00270 123* SUM = SUM-N(J)
00271 124* 40 CONTINUE
00273 125* 45 IF (TAU .GE. T) GO TO 55
00275 126* TMI = T-1
00276 127* DO 50 J=TAU,TMI
00301 128* SUM = SUM-N(J)
00302 129* 50 CONTINUE
00304 130* 55 IF (SUM .GT. LX1(T)) LX1(T) = SUM
00306 131* 60 CONTINUE
00310 132* MX1(T) = WM-N(T)
00311 133* IF (T .EQ. TTPI) GO TO 63
00313 134* TPI = T+1
00314 135* DO 64 TAU=TTPI,TTPI
00317 136* SUM = N(TAU)
00320 137* TAJM1 = TAU-1
00321 138* DO 52 J=T,TAJM1
00324 139* SUM = SUM-N(J)
00325 140* 62 CONTINUE
00327 141* IF (WM-SUM .LT. MX1(T)) MX1(T) = WM- SUM
00331 142* 64 CONTINUE
00333 143* 69 IF (LX1(T) .LE. MX1(T)) GO TO 70
00335 144* WRITE (6,940) T,LX1(T),MX1(T)
00342 145* 940 FORMAT (////, 'ERROR - THE PROBLEM IS INFEASIBLE. FOR YEAR T = ',
00342 146* '13', ' LAMBDA(X1T) = ',13, ' IS GREATER THAN MU(X1T) = ',13)
00343 147* STOP
00344 148* 70 CONTINUE
00346 149* IF (LX1(1) .EQ. WM .AND. MX1(1) .EQ. WM) GO TO 71
00350 150* WRITE (6,945) LX1(1),MX1(1),WM
00355 151* 945 FORMAT(////, 'ERROR - THE PROBLEM IS INFEASIBLE. FOR YEAR T = 1, LA
00355 152* 'MBDA(X1T) = ',13, ' AND MU(X1T) = ',13, ' BUT THE INITIAL FLEET SIZE
00355 153* ' IS WM = ',13)
00356 154* STOP
00356 155*
00356 156* C SUBROUTINE X2LIM CALCULATES LIMITS ON X2T.
00356 157* C X2T IS THE NUMBER OF RESOURCES PURCHASED PRIOR TO YEAR T.
00356 158* C MX2(T) IS THE UPPER LIMIT OF X2T IN YEAR T.
00356 159* C :X1(T) IS THE NUMBER OF VALUES X1T ASSUMES THROUGH YEAR T.

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00036 160* L2=X1(J) IS THE LOWER LIMIT OF X2T GIVEN THE J-TH VALUE OF X1T.
00039 161* C A1(1)=X1(1) IS THE FIRST VALUE OF X1T, X1(2) THE SECOND,
00040 162* C A1(2)+1 THE THIRD, ETC.
00041 163*
00042 164*
00043 165*
00044 166*
00045 167*
00046 168*
00047 169*
00048 170*
00049 171*
00050 172*
00051 173*
00052 174*
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210* 00450 1BEST = -2**32
219* 00451 DO 300 IAUJ=1,ITP1
220* 00454 B15 = 0
221* 00455 IF (IAU .GT. T) GO TO 295
222* 00457 DO 295 JTAU,T
223* 00462 B15 = B15-NU(J)
224* 00463 295 CONTINUE
225* 00465 IF (IAU .GT. 1) GO TO 295
226* 00467 B15 = B15+NU
227* 00470 GO TO 298
228* 00471 298 IAUJ1 = IAUJ-1
229* 00472 IPI = T+1
230* 00473 B15 = B15+(TAUJ1)
231* 00474 IF (IAU .LE. TPI) GO TO 298
232* 00476 DO 297 JETPI,TAUJ1
233* 00501 B15 = B15-NU(J)
234* 00502 297 CONTINUE
235* 00504 292 IF (B15 .GT. BIGEST) BIGEST=B15
236* 00506 300 CONTINUE
237* 00510 MD1 = MIN(MD1,X1T+X2T+M(T)-BIGEST)
238* 00511 LD1 = MAX(0,X1T-MX1(T+1))
239* 00512 MD2 = 4(T)
240* 00513 WRITE (6,966) L3,M3,LD1,MD1,MD2
241* 00522 966 FORMAT (' L3=,I10,, M3=,I10,, LD1=,I10,, MD2=,I10,')
242* 00523 IF (LD1 .LE. MD1) GO TO 301
243* 00525 WRITE (6,946) T,X1T,X2T,LD1,MD1
244* 00534 946 FORMAT (////, ' ERROR - THE PROBLEM IS INFEASIBLE. FOR YEAR T = ,I3,')
245* 00534 **, ' WITH X1T = ,I3,, AND X2T = ,I3,, LAMBDA(DIT) = ,I3,, IS AREA
246* 00534 **, ' LARGER THAN MU(DIT) = ,I3,'
247* 00535 STOP
248* 00536 301 DO 140 X3T=L3,M3,J1
249* 00541 ICOUNT = ICOUNT+1
250* 00542 IF (ICOUNT .LE. STATE) GO TO 125
251* 00544 STATE = STATE+1
252* 00545 WRITE (6,999) T,STATE1
253* 00551 999 FORMAT (////, ' ERROR - IN YEAR T = ,I3,, THERE ARE MORE THAN STATE
254* 00551 **, ' STATES. INCREASE THE VALUE OF STATE ON THE PARAMETER CAR
255* 00551 **, 'DS', ' IN THE MAIN PROGRAM AND ALL SUBROUTINES. ')
256* 00552 STOP
257* 00553 125 KINC = (X3T-L3)/J1
258* 00554 IF (KINC .GT. 0) GO TO 132
259* 00555 RTBEST = 2**35
260* 00557 DO 130 D1=LJ1,MD1
261* 00562 IATPI = X1T-D1T-LX1(T+1)+MX1(T)+1
262* 00563 LD2=MAX(0,LX2X1(IATPI)-X2T)
263* 00564 IF (LD2 .LE. MD2) GO TO 127
264* 00566 WRITE (6,947) T,X1T,X2T,D1T,LD2,MD2
265* 00576 947 FORMAT (////, ' ERROR - THE PROBLEM IS INFEASIBLE. FOR YEAR T = ,I3,')
266* 00576 **, ' WITH X1T = ,I3,, X2T = ,I3,, AND D1T = ,I3,, LAMBDA(D2T) =
267* 00576 **, ' IS', ' GREATER THAN MU(D2T) = ,I3,'
268* 00577 STOP
269* 00580 127 DO 130 D2=LJ2,MD2
270* 00603 CALL STGR1
271* 00604 IPI = 0
272* 00605 IF (T .LT. TP) CALL TRNFM
273* 00607 RT = FLOAT(RN)+DELTA*FLOAT(RTP1)+5
274* 00610 WRITE (6,932) X1T,X2T,X3T,D1T,D2T,RN,RTP1,RT, TBEST,D1BEST,D2BEST
275* 00625 932 FORMAT (2I5,2I5,2I5,4I15,2I5)

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00450 210* BIGEST = -2**32
00451 00 300 TAJ=1,IT=1
00454 220* BIG = 0
00455 241* IF (TAJ.EQ. 1) GO TO 295
00457 222* DO 295 J=TAU,1
00462 223* BIG = BIG-VN(J)
00463 224* CONTINUE
00465 225* IF (TAU.EQ. 1) GO TO 295
00467 226* BIG = BIG+VW
00470 227* GO TO 298
00471 228* TAJ=1 = TAJ-1
00472 229* TPI = T+1
00473 230* BIG = BIG+J(TAJ+1)
00474 231* IF (TAU.LE. TPI) GO TO 298
00476 232* DO 297 J=TPI,TAJW1
00501 233* BIG = BIG-V(J)
00502 234* CONTINUE
00504 235* IF (BIG.EQ. BIGEST) BIGEST=BIG
00506 236* CONTINUE
00510 237* VJ1 = MIN(WJ1,X1T+X2T+W(T)-BIGEST)
00511 238* LJ1 = MAX(0,X1T-WX1(T+1))
00512 239* VJ2 = V(T)
00513 240* WRITE (6,966) L3,W3,LJ1,WJ1,WJ2
00522 241* IF (L3.EQ. 1) GO TO 301
00523 242* IF (LJ1.LE. WJ1) GO TO 301
00525 243* WRITE (6,946) T,X1T,X2T,LJ1,WJ1
00534 244* FORMAT (////, ERROR - THE PROBLEM IS INFEASIBLE. FOR YEAR T = ,I3
00534 245* *, WITH X1T = ,I3, AND X2T = ,I3, LAMBDA(DIT) = ,I3, IS AREA
00534 246* *TER,*, THAN MU(DIT) = ,I3)
00535 247* STOP
00536 248* DO 140 X3T=L3,W3,J1
00541 249* ICOUNT = ICOUNT+1
00542 250* IF (ICOUNT.LE. STATE) GO TO 125
00544 251* STATE = STATE
00545 252* WRITE (6,999) T,STATE1
00551 253* FORMAT (////, ERROR - IN YEAR T = ,I3, THERE ARE MORE THAN STATE
00551 254* * = ,I6, STATES. INCREASE THE VALUE OF STATE ON THE PARAMETER CAR
00551 255* *DS,, IN THE MAIN PROGRAM AND ALL SUBROUTINES.)
00552 256* STOP
00553 257* KINC = (X3T-L3)/J1
00554 258* IF (X1C.EQ. n) GO TO 132
00555 259* BIGEST = 2**33
00557 260* DO 130 J1=LJ1,WJ1
00562 261* IATPI = X1T-J1T-LX1(T+1)+VX1(T)+1
00563 262* LJ2=MAX(0,LX2X1(IX1TPI)-X2T)
00564 263* IF (LJ2.LE. WJ2) GO TO 127
00566 264* WRITE (6,947) T,X1T,X2T,J1T,LJ2,WJ2
00576 265* FORMAT (////, ERROR - THE PROBLEM IS INFEASIBLE. FOR YEAR T = ,I3
00576 266* *, WITH X1T = ,I3, X2T = ,I3, AND J1T = ,I3, LAMBDA(D2T) =
00576 267* *,I3, IS,*, GREATER THAN MU(D2T) = ,I3)
00577 268* STOP
00600 269* DO 130 J2=LJ2,WJ2
00603 270* CALL STGR=1
00604 271* IF (T.LT. 1) CALL TR=VW
00605 272* RT = FLOAT(9N)+DELTA*FLOAT(RTP1)+.5
00607 273* WRITE (6,942) VIT,X2T,J3T,DIT,D2T,RN,RTD1,RT, TBEST,DIREST,D2REST
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01005 334* X2T = 0
01006 335* X3T = 0
01007 336* F1(1) = F1(1)/100
01008 337* WRITE (6,950) ST(1)
01009 338* FORMAT (1) THE TOTAL COST OF THE FOLLOWING EQUIPMENT REPLACEMENT 90
01010 339* *CITY IS 5, 115, //, YEAR SELL BUY *)
01011 340* WRITE (6,955) T,D,(1),RV(2)
01012 341* FORMAT (14,19,110)
01013 342* DIT = DN(1)
01014 343* D2T = DN(2)
01015 344* ITW1 = IT-1
01016 345*
01017 346* THE FORWARD PASS OF THE DYNAMIC PROGRAM BEGINS HERE.
01018 347*
01019 348* DO 220 I=1,ITW1
01020 349* READ (10UT) TPI,N0,ICOUNT
01021 350* READ (10UT) JNDX1(I),I=1,N0),S,M
01022 351* READ (10UT) JNDX2(I),I=1,SUM)
01023 352* I02 = 2*ICOUNT
01024 353* READ (10UT) JN(I),I=1,I02)
01025 354* CALL TRANSF
01026 355* DIT = DN(INDEX*2-1)
01027 356* D2T = DN(INDEX*2)
01028 357* X1T = Y1TP1
01029 358* X2T = Y2TP1
01030 359* X3T = Y3TP1
01031 360* WRITE (6,955) TPI,DIT,D2T
01032 361* IF (T.EQ. ITW1) STOP
01033 362* DO 205 I=1,8
01034 363* BACKSPACE 10UT
01035 364* CONTINUE
01036 365* 220 CONTINUE
01037 366* END
01038 367* SUBROUTINE STORST
01039 368* 1*
01040 369* 2* STORST CALCULATES THE STAGE RETURN.
01041 370* 3*
01042 371* 4* IMPLICIT INTEGER (A-H,O-Z)
01043 372* 5* REAL DELTA
01044 373* 6* PARAMETER NRS=25, NP05=100, MAXAGE=25, STATE=10000
01045 374* 7* PARAMETER JPOS=NP05/4, MP05=NP05*2, NRS1=NRS+1, STATE2=2*STATE
01046 375* 8* COMMON M(N,NRS),P(NRS),LV1(NRS1),NX1(NRS1),LX21(N0X1),RN,DELTA,
01047 376* 9* 4X2(NRS1),FTPL(STATE),IS(NPCS),D(NRPS),JNDX1(NP05),J0,J1,
01048 377* 10* JNDX2(MP05),FLAG,RTP1,INDEX,V(MAXAGE,NRS),NX1(NRPS1),IT,
01049 378* 11* NUN(NRS),Q(MAXAGE),N(NRPS1),FT(STATE),JNDX1(NP05),
01050 379* 12* INDX2(MP05),DN(STATE2),VM,N0X11
01051 380* 13* COMMON X1T,X2T,X3T,DIT,D2T,Y1TP1,Y2TP1,Y3TP1,
01052 381* 14* RN=U
01053 382* 15* L=FT=X1T-DIT
01054 383* 16* IF (LEFT,LT,1) GO TO 20
01055 384* 17* DO 10 I=1,LEFT
01056 385* 18* R4 = RN+J0+(IS(I)+T)*U1
01057 386* 19* 20 CONTINUE
01058 387* 21*
01059 388* 22* IF (J1T,LT,1) GO TO 40
01060 389* 23* DO 30 I=1,DIT
01061 390* 24* J=IS(I)+LEFT)
01062 391* 25* R4 = RN-V(J,I)
01063 392* 26* CONTINUE
01064 393* 27* IF (D2T,GT,0) RN=RN+(P(T)+J0+U1)*D2T
01065 394* 28* IF (X2T,GT,0) RN=RN+X3T+J1*X2T
01066 395* 29* RETURN
01067 396* 30* END
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00101 1* SUBROUTINE TRANSFM
00101 2*
00101 3* TRANSFM USES THE STAGE TRANSFORMATION TO PICK UP THE CORRECT VALUE
00101 4* OF THE MINIMAL COST FROM STAGE I+1 TO IT, RTPI.
00101 5*
00103 6*
00104 7* IMPLICIT INTEGER (A-H,O-Z)
00105 8*
00106 9* REAL DELTA
00107 10*
00108 11* PARAMETER NPOS=25,NPCS=100,MAXAGE=25,STATE=10000
00109 12*
00110 13* PARAMETER NPOS=NPCS/4,MPOS=NPOS**2,NYRS1=NYRS+1,STATE2=2*STATE
00111 14*
00112 15* PARAMETER NOX1=NYRS1*NPCS
00113 16*
00114 17* COMMON /NYRS1,P(NYRS),LX1(NYRS1),MX1(NYRS1),LX2X1(NOX1),R1,DELTA,
00115 18* MX2(NYRS1),FTPI(STATE),IS(NPCS),J(NYRS),JNOX1(NPOS),JO,II,
00116 19* JNOX2(NPOS),FLAG,RTPI,INDEX,V(MAXAGE,NYRS),NX1(NYRS1),IT,
00117 20* VN(NYRS),Q(MAXAGE),V(NYRS1),FT(STATE),INDX1(NPOS),
00118 21* INJX2(MPOS),DN(STATE2),VM,NOX11
00119 22*
00120 23* COMMON X1T,X2T,X3T,D1T,D2T,Y1TP1,Y2TP1,Y3TP1,T
00121 24*
00122 25* INDEX = 0
00123 26*
00124 27* Y1TP1 = X1T-D1T
00125 28*
00126 29* Y2TP1 = X2T-D2T
00127 30*
00128 31*
00129 32* Y3TP1=X3T+J1*X2T+(J0+U1)*D2T
00130 33*
00131 34* I1 = Y1TP1-LX1(I+1)
00132 35*
00133 36* I2=0
00134 37*
00135 38* IF (I1.LT.1) GO TO 20
00136 39*
00137 40* I3=0
00138 41*
00139 42* DO 10 J=1,I1
00140 43*
00141 44* I2=I3+1
00142 45*
00143 46* I3=JNOX1(J)+I3
00144 47*
00145 48* DO 10 K=I2,I3
00146 49*
00147 50* IINDEX = INDEX+JNOX2(K)
00148 51*
00149 52* 10 CONTINUE
00150 53*
00151 54* L2 = NX1(I)+Y1TP1-LX1(I+1)+1
00152 55*
00153 56* L2 = LX2X1(L2)
00154 57*
00155 58* I2=Y2TP1-L2
00156 59*
00157 60* IF (I2.LT.1) GO TO 40
00158 61*
00159 62* I4=I3+1
00160 63*
00161 64* I3=I3+I2
00162 65*
00163 66* DO 30 J=I4,I3
00164 67*
00165 68* IINDEX=INDEX+JNOX2(J)
00166 69*
00167 70* 30 CONTINUE
00168 71*
00169 72* TPI=T+1
00170 73*
00171 74* FLAG=1
00172 75*
00173 76* CALL X3LIM(TPI,Y1TP1,Y2TP1,L3,M3)
00174 77*
00175 78* INDEX = INDEX+(Y3TP1-L3)/U1+1
00176 79*
00177 80* RTPI=FTPI(INDEX)
00178 81*
00179 82* RETURN
00180 83*
00181 84* END
00182 85*
00183 86* SUBROUTINE X3LIM (T,X1-HAT,X2-HAT,L3,M3)
00184 87*
00185 88* X3LIM CALCULATES BOUNDS L3 AND M3 ON X3T IN STAGE T, GIVEN X2T.
00186 89*
00187 90*
00188 91* IMPLICIT INTEGER (A-H,O-Z)
00189 92*
00190 93* REAL DELTA
00191 94*
00192 95* PARAMETER NYRS=25,NPCS=100,MAXAGE=25,STATE=10000
00193 96*
00194 97* PARAMETER NPOS=NPCS/4,MPOS=NPOS**2,NYRS1=NYRS+1,STATE2=2*STATE
00195 98*
00196 99* PARAMETER NOX1=NYRS1*NPCS
00197 100*
00198 101* COMMON /NYRS1,P(NYRS),LX1(NYRS1),MX1(NYRS1),LX2X1(NOX1),R1,DELTA,
00199 102* MX2(NYRS1),FTPI(STATE),IS(NPCS),J(NYRS),JNOX1(NPOS),JO,II,
00200 103* JNOX2(NPOS),FLAG,RTPI,INDEX,V(MAXAGE,NYRS),NX1(NYRS1),IT,
00201 104* VN(NYRS),Q(MAXAGE),V(NYRS1),FT(STATE),INDX1(NPOS),
00202 105* INJX2(MPOS),DN(STATE2),VM,NOX11
00203 106*
00204 107* DIMENSION X2BAR(NYRS1),X1STAR(NYRS1),X2STAR(NYRS1),X2TIL(J(NYRS1)
00205 108*
00206 109*
00207 110*
00208 111*

```

```

00112 10* 3=U
00113 11* T=1-I
00114 12* DO 30 TAU=1,T
00117 13* IF (TAU.GT. 1) GO TO 30
00121 20* SUM=0
00122 21* DO 10 J=1,TM1
00125 22* SUM=SUM+M(J)
00126 23* 1) CONTINUE
00130 24* 20 IF (TAU.GT. 1) SUM=SUM-I*(TAU-1)
00132 25* X1STAR(TAU)=MIN(X1(TAU),X1HAT+SUM)
00133 26* 3) CONTINUE
00135 27* DO 40 TAU=1,T
00140 28* X2STAR(TAU)=-2**33
00141 29* DO 50 SIGMA=1,T
00144 30* SIG=AM-X1STAR(SIGMA)
00145 31* IF (SIGMA.GT. 1) SIG=0(SIGMA-1)-V1STAR(SIGMA)
00147 32* IF (SIGMA.LE. TAU) GO TO 50
00151 33* SIG-1=SIGMA-1
00152 34* DO 40 J=TAU,SIGM1
00155 35* SIG=SIG-M(J)
00156 36* 4) CONTINUE
00160 37* 5) IF (SIG.GT. X2STAR(TAU)) X2STAR(TAU)=SIG
00162 38* 6) CONTINUE
00164 39* IF (TAU.GT. 1) GO TO 40
00166 40* SUM=X2HAT
00167 41* DO 70 J=1,TM1
00172 42* SUM=SUM-M(J)
00173 43* 7) CONTINUE
00175 44* 8) IF (TAU.GT. 1) SUM=SUM+M(TAU-1)
00177 45* X2BAR(TAU)=MAX(X2STAR(TAU),SUM)
00200 46* 9) CONTINUE
00202 47* DO 100 TAU=1,TM1
00205 48* L3=L3+(J0+J1*(T-TAU))*(X2BAR(TAU+1)-X2BAR(TAU))
00210 49* 100) CONTINUE
00212 50* IF (FLAG.EQ. 1) RETURN
00212 51* IF (T.EQ. 2) WRITE (6,900) T,X1HAT,X2HAT,(X1STAR(J),X2STAR(J),
00212 52* *X2BAR(J),J=1,T),L3
00227 53* 900 FORMAT (9I5,I15)
00230 54* SUM=0
00231 55* DO 110 TAU=1,T
00234 56* IF (TAU.GT. 1) SUM=SUM+M(TAU-1)
00236 57* X2TILDE(TAU)=X2HAT
00237 58* IF (X2HAT.LE. SUM) X2TILDE(TAU)=SUM
00241 59* 11) CONTINUE
00243 60* L3=L3+(J0+J1*(T-TAU))*(X2TILDE(TAU+1)-X2TILDE(TAU))
00244 61* DO 120 TAU=1,TM1
00247 62* V3=V3+(J0+J1*(T-TAU))*(X2TILDE(TAU+1)-X2TILDE(TAU))
00250 63* 12) CONTINUE
00252 64* IF (L3.LE. V3) RETURN
00254 65* WRITE (6,905) T,X1HAT,X2HAT,L3,M,T,X1,X2,X1HAT,X2HAT
00253 66* 905 FORMAT (////,ERROR - THE PROBLEM IS INFEASIBLE. FOR YEAR T=,I3
00253 67* *,WITH X1=,I15,X2=,I15, AND XPT=,I3, LAMBDA(XPT)=,E12, IF GRE
00253 68* *ATER, THEN MAX(XPT)=,I12)
00254 69* STOP
00255 70* END

```

```

00101 1* SUBROUTINE X2LIM
00101 2*
00101 3* X2LIM CALCULATES 30UNJS ON X2T GIVEN EVERY POSSIBLE VALUE OF X1T
00101 4* FOR ALL STAGES.
00101 5*
00101 6* IMPLICIT INTEGER (A-H,O-Z)
00101 7*
00101 8* REAL DELTA
00101 9*
00101 10* PARAMETER NPOS=25,NPCS=100,MAXAGE=25,STATE=10000
00101 11* PARAMETER NPOS=NPCS/4,NPOS=NPOS**2,NYRS1=NYRS+1,STATE2=2*STATE
00101 12* PARAMETER NOX1=NYRS1*NPCS
00101 13*
00101 14* COMMON M(NYRS),P(NYRS),LX1(NYRS1),MX1(NYRS1),LX2X1(NOX1),R1,DELTA,
00101 15* MX2(NYRS1),FTPL1(STATE),IS(NPCS),D(NYRS),JNY1(NPOS),JO11,
00101 16* UNX2(NPOS),FLAG,RTPL,INDEX,V(MAXAGE,NYRS),NX1(NYRS1),TT,
00101 17* NN(NYRS),O(MAXAGE),N(NYRS1),FT(STATE),INDX1(NPOS),
00101 18*
00101 19* INX2(NPOS),ON(STATE2),MM,NOX11
00101 20*
00101 21* COMMON X1T,X2T,X3T,D1T,D2T,Y1TPL,Y2TPL,Y3TPL,T
00101 22* TTPL = TT+1
00101 23* ICOUNT = 0
00101 24*
00101 25* DO 50 TEL,TTPL
00101 26* IF (T.GT. 1) MX2(T) = MX2(T-1)+M(T-1)
00101 27* L1 = LX1(T)
00101 28* M1 = MX1(T)
00101 29* DO 40 X1T=L1,M1
00101 30* LAM1 = 0
00101 31* DO 30 TAU=2,TTPL
00101 32* SMALL = X1T
00101 33* IF (TAU .GE. T) GO TO 15
00101 34* TM1 = T-1
00101 35* DO 10 JETAU,TM1
00101 36* SMALL = SMALL+NN(J)
00101 37*
00101 38* 10 CONTINUE
00101 39* 15 LAM1 = D(TAU-1)-MIN(MX1(TAU),SMALL)
00101 40* IF (TAU .LE. T) GO TO 25
00101 41* TAU1 = TAU-1
00101 42* DO 20 JET,TAU1
00101 43* LAM1 = LAM1-M(J)
00101 44*
00101 45* 20 CONTINUE
00101 46* 25 IF (LAM1 .GT. LAM1) LAM1 = LAM1
00101 47* 30 CONTINUE
00101 48* IF (LAM1 .LE. MX2(T)) GO TO 38
00101 49* WRITE (6,900) T,X1T,MX2(T),LAM1
00101 50* 900 FORMAT (////, 'PROBLEM IS INFEASIBLE. FOR T = ',I2,' AND X1T = ',
00101 51* 'I3,' 'M(X2T) = ',I3,' AND LAM1D(X2T) = ',I3)
00101 52* STOP
00101 53* 38 ICOUNT = ICOUNT+1
00101 54* IF (ICOUNT .LE. NOX11) GO TO 30
00101 55* WRITE (6,910)
00101 56* 910 FORMAT (////, 'ERROR - X1T ASSUMES MORE THAN NOX1 VALUES. INCREASE
00101 57* THE VALUE OF NOX1 ON THE PARAMETER CARDS IN THE MAIN PROGRAM AND',
00101 58* ' ALL SUBROUTINES.')
00101 59* STOP
00101 60* 39 LAM2X1(ICOUNT) = LAM1
00101 61* 40 CONTINUE
00101 62* X1(T) = ICOUNT
00101 63* 50 CONTINUE
00101 64* I=NX1(TTPL)
00101 65* WRITE (6,905) (LX2X1(J),J=1,I)
00101 66* 905 FORMAT (25I5)
00101 67* END
00101 68*
00101 69* B-10
00101 70*

```

APPENDIX C

AN INTEGER PROGRAMMING MODEL

The model described in this Appendix is a somewhat simplified integer programming (IP) analog to the DP model presented in Section 3. Since the IP version is subsumed under the DP version, the former is documented here for its own sake, as an application of integer programming, and is not necessarily intended to serve as an "alternative" model.

As in the DP model, the IP model prescribes actions to be taken each year for a T-year period to minimize the total cost over those T years. From a given initial fleet, the decisions specify the number of purchases each year and the number of retirements, from the initial fleet, of engines of each age \underline{a} . (Note that T may not be taken so large as to make liable to retirement engines which were purchased during the T-year period.) These decisions are to be made so as to minimize the total cost for the T years, subject to the constraint that a specified minimum fleet size be met each year.

The variables are:

x_{at} = the number of engines, initially of age \underline{a} , retired in year t,

y_t = the number of new engines purchased in year t.

The conventions regarding age definition and decision times are the same as for the DP model (cf., footnote 6).

The data required by the model include:

D_t = the minimum number of engines required during year t
(checked against the fleet size after year t 's decisions
have been made),

M_t = the maximum number of engines which may be purchased in
year t ,

P_t = the purchase price of an engine in year t ,

Q_a = the number of a - year - old engines in the initial fleet,

u_a = the maintenance cost of an engine during its a^{th} year of
service,

v_{at} = the resale value in year t of an engine which was initially
of age a .

Note that this model does not have a ceiling on the number of engines that may be retired, nor does it have a mandatory retirement age, as does the DP model. If a set A of ages of engines in the initial fleet is given, then the model requires data for u and v for a as large as $\mu + T$, where μ is the maximum age in A .

Using the above definitions, the IP is formulated as:

minimize

$$\sum_{t=1}^T \{P_t + u_1\}y_t + \sum_{a \in A} [u_{a+t} (Q_a - \sum_{\tau \leq t} x_{a\tau}) - v_{at}x_{at}] + \sum_{\tau < t} u_{t-\tau+1} y_{\tau} \quad (C-1)$$

¹⁴ The term $\sum_{t=1}^T \sum_{a \in A} u_{a+t} Q_a$ in the objective function (C-1) does not affect the minimizing values of x_{at} and y_t , but it must be included to calculate the minimum value of (C-1). Also, discounting has been omitted for simplicity and could clearly be implemented in the model.

subject to

$$\sum_{t=1}^T x_{at} \leq Q_a \quad (a \in A), \quad (C-2)$$

$$y_t \leq M_t \quad (t=1, \dots, T), \quad (C-3)$$

$$\sum_{a \in A} Q_a + \sum_{\tau \leq t} (y_\tau - \sum_{a \in A} x_{a\tau}) \geq D_t \quad (t=1, \dots, T) \quad (C-4)$$

$$x_{at}, y_t \text{ nonnegative integers } (a \in A, t=1, \dots, T) \quad (C-5)$$

The expressions $\{ \}$ summed in (C-1) are the costs for the individual years t . Each of these is calculated from the following components:

$(P_t + u_1)$ = the cost of purchasing an engine and maintaining it during its first year of service,

$u_{a+t}(Q_a - \sum_{\tau \leq t} x_{a\tau})$ = the maintenance cost in year t of engines,

initially of age \underline{a} , which remain in the fleet,

$v_{at}x_{at}$ = the revenue from retiring x_{at} engines, initially of age \underline{a} , in year t ,

$\sum_{\tau < t} u_{t-\tau+1}y_\tau$ = the maintenance cost in year t of engines purchased during years $\tau = 1, \dots, t-1$.

Constraint (C-2) specifies that the total number of engines retired, initially of age \underline{a} , not exceed the initial number of age \underline{a} engines, and constraint (C-3) restricts to at most M_t the number of engines purchased in year t . Constraint (C-4) requires that the number of engines in the fleet in year t (after purchases and retirements

in year t) to be at least D_t . If d is the number of distinct ages in the set A of ages, then the IP in (C-1) through (C-5) has $d + 2T$ constraints and $(d + 1)T$ variables.

The reader may have observed that the IP described above does not specify any retirement order. The condition that engines be retired in order of decreasing age may be imposed by the following suggestion of A. J. Goldman. This uses $(d - 1)T$ additional variables and $2(d - 1)T$ additional constraints :

$$\sum_{\alpha < a} x_{\alpha t} \leq \left(\sum_{\alpha < a} Q_{\alpha} \right) \delta_{at}, \quad (C-6)$$

$$Q_a - \sum_{\tau \leq t} x_{a\tau} \leq Q_a (1 - \delta_{at}), \quad (C-7)$$

with $a \in A$, $a \neq \min \{\alpha | \alpha \in A\}$ and $t = 1, \dots, T$. The 0-1 variable δ_{at} acts as a "switch": if $\delta_{at} = 0$, then the retiring of engines of initial age less than \underline{a} in year t is prohibited by (C-6), and (C-7) is non-constraining, whereas if $\delta_{at} = 1$ such engines may be retired since (C-7) together with (C-2) would imply that all Q_a engines, initially of age a , have been retired, and the right side of (C-6) is non-constraining in view of (C-2). Of course, the constraints (C-6) and (C-7) may be introduced only as they are needed. Thus if the IP (C-1) - (C-5) yields a solution in which retirements are partially "out of order," (C-6) and (C-7) would be imposed only for the exceptional pairs (a, t) . The nature the solution will depend on the data, and if these are "reasonable" one might expect the "order"

condition to hold on its own.

